Math in Nature:

Finding order in chaos

Jessica Kunke
Lei Zhu
Stephen Portillo

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Seminar Outline

Math models and nonlinearity
Jessica

Butterfly effect and weather forecasting
Lei

Structure in chaos
Stephen

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Part 1:
Math models and nonlinearity

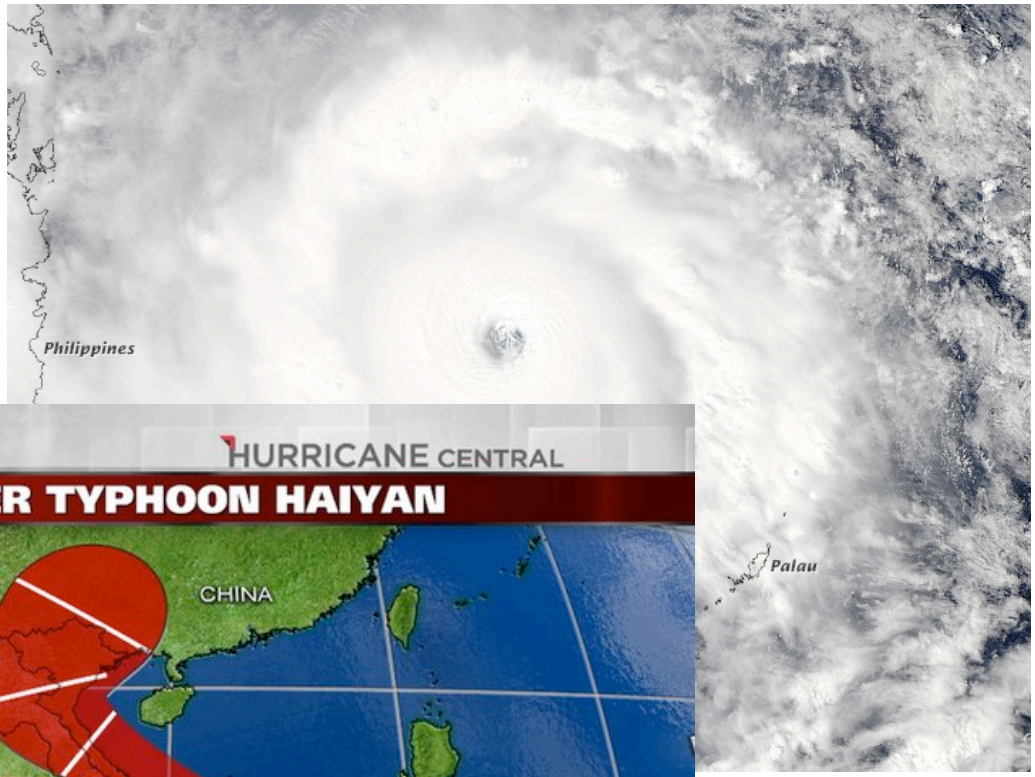
Jessica Kunke

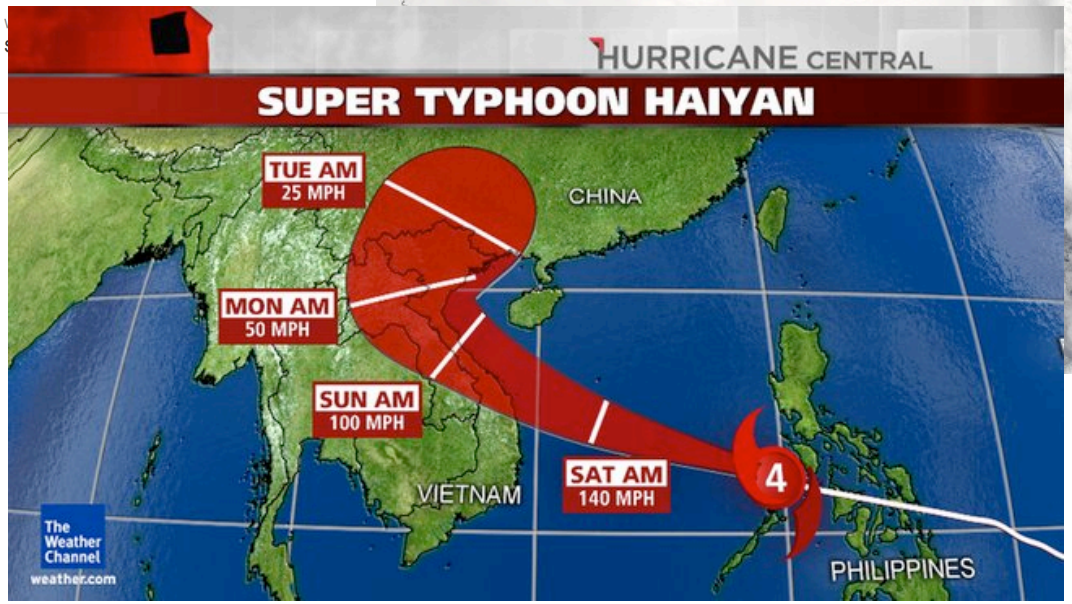
Outline

- Modeling our world
- Linear or nonlinear? That is the question...

Why model? To *predict*

Wed Nov 13	Sunny	39° 28°	CHANCE OF PRECIP: 0%	WIND: NW at 16 mph Details
Thu Nov 14	Sunny	50° 31°	CHANCE OF RAIN: 0%	WIND: SW at 15 mph Details
Fri Nov 15	Mostly Cloudy	56° 35°	CHANCE OF RAIN: 0%	WIND: SW at 10 mph Details
Sat Nov 16	AM Clouds / PM Sun	55° 36°	CHANCE OF RAIN: 0%	





http://www.weather.com/weather/tenday/Cambridge+MA+02140:4:US
http://www.weather.com/weather/hurricanecentral/
http://earthobservatory.nasa.gov/NaturalHazards/view.php?id=82341

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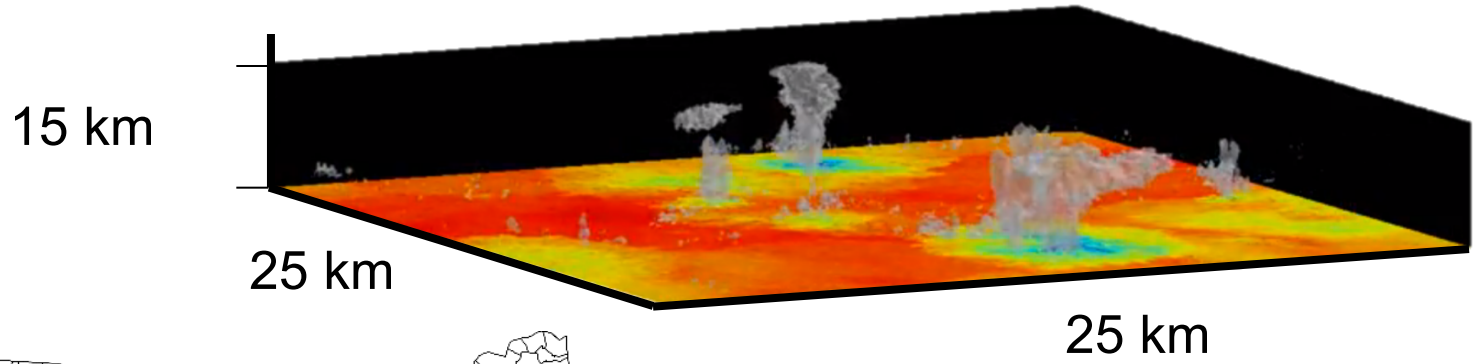
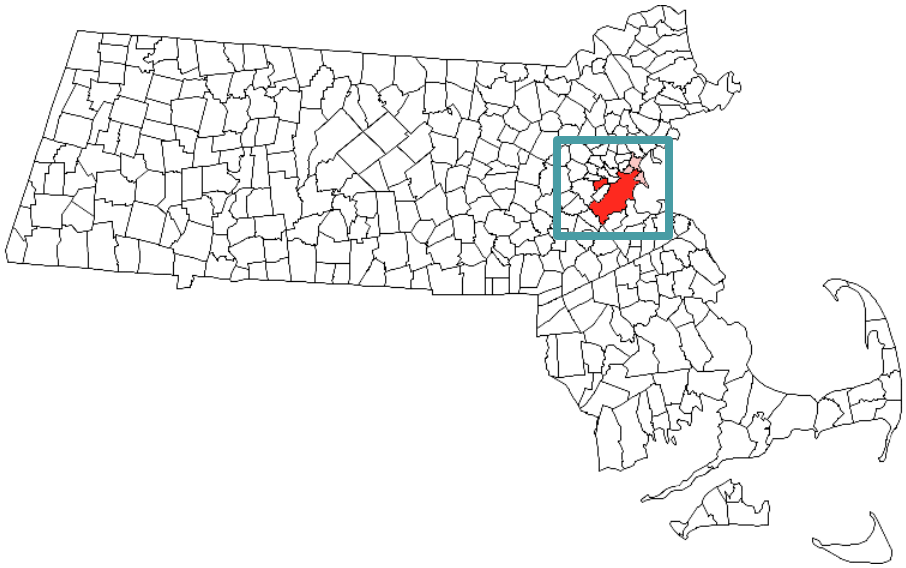
Why model? To *understand*





~3x the area of Boston

http://en.wikipedia.org/wiki/File:Boston_ma_highlight.png
http://www.youtube.com/watch?v=kLUzQjE8wU4

What's in a model?

- equations (relationships)
- not exact reality, but useful
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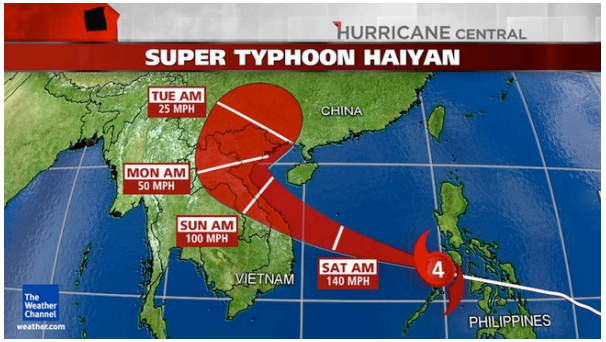
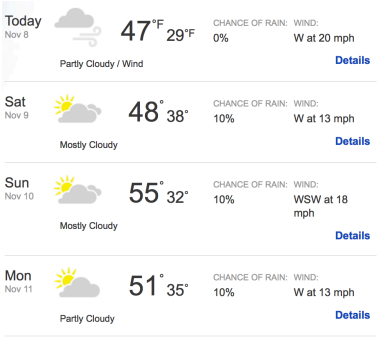
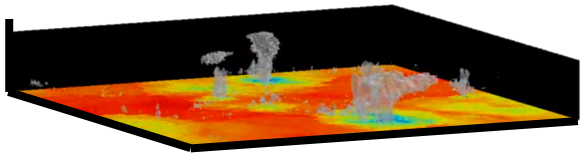
  - to simulate ≠ to understand
- involves making simplifications
- no single model
- can use observations
- might use computers







http://www.weather.com/weather/tenday/Cambridge+MA+02140:4:US
http://www.weather.com/weather/hurricanecentral/
http://www.youtube.com/watch?v=kLUzQjE8wU4

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An example: crop yield

light?

water?

temperature?

soil composition?

pollution?

pests?

wind?

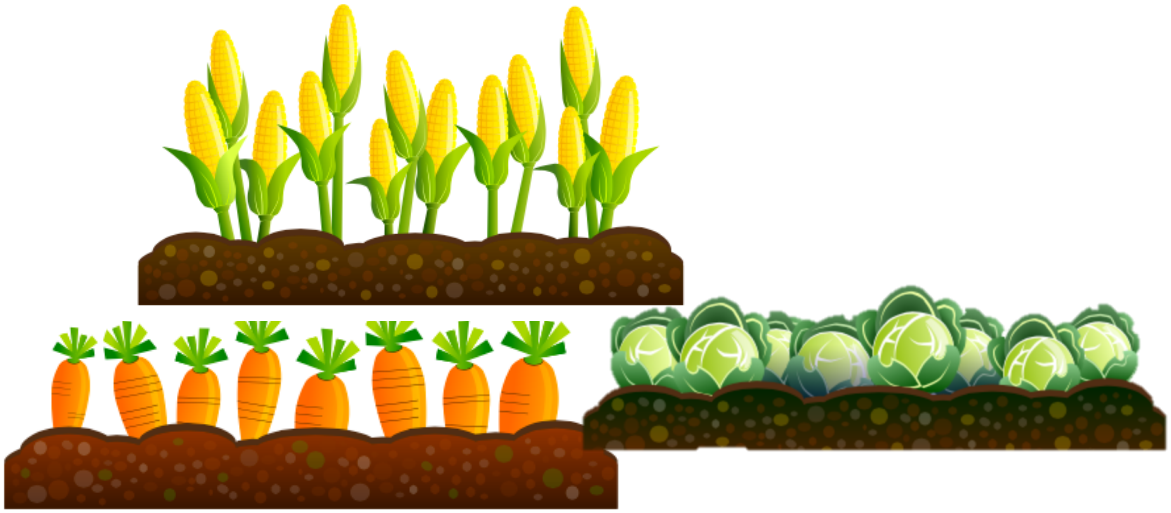
genetics?

management?

weeds?

crop rotation?

...other?



http://openclipart.org/detail/168724/simple-farm-crops-by-viscious-speed

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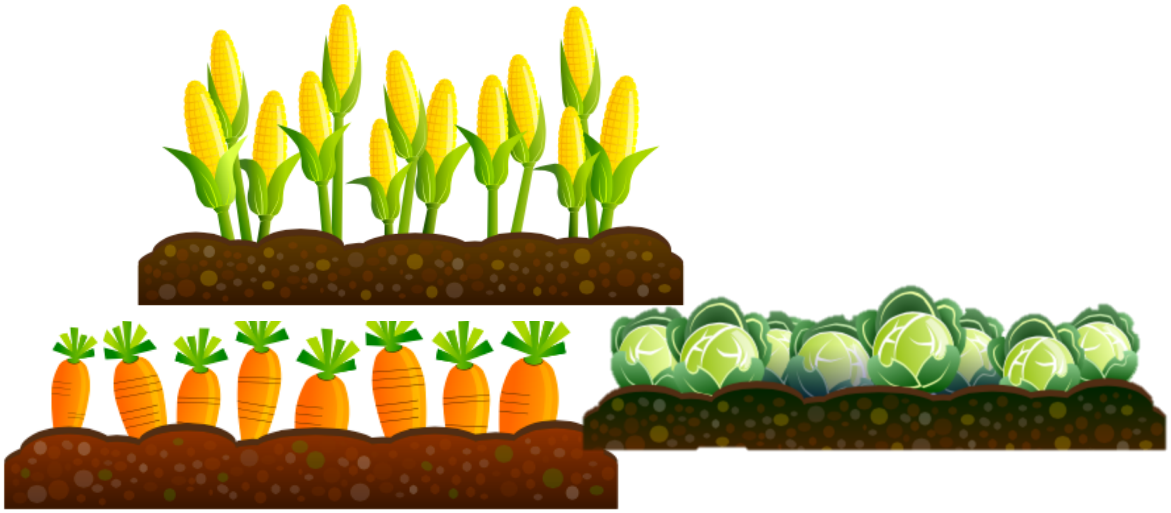

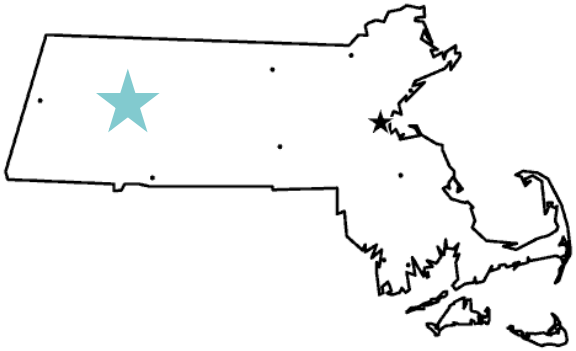

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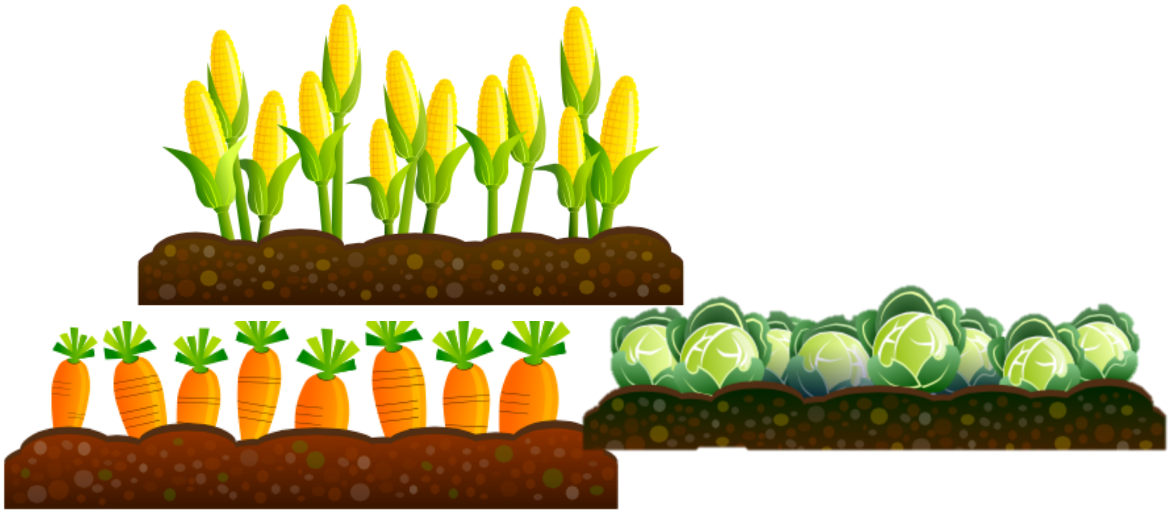

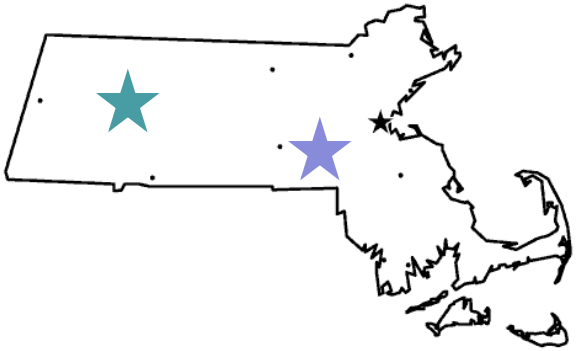

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An example: crop yield

light

water

temperature

soil composition

color

duration

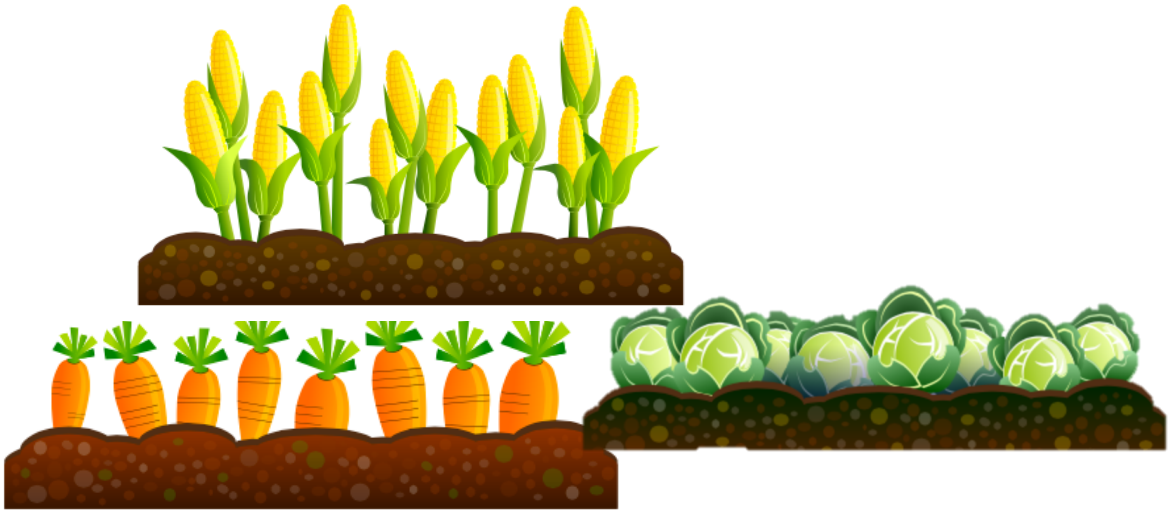
pests

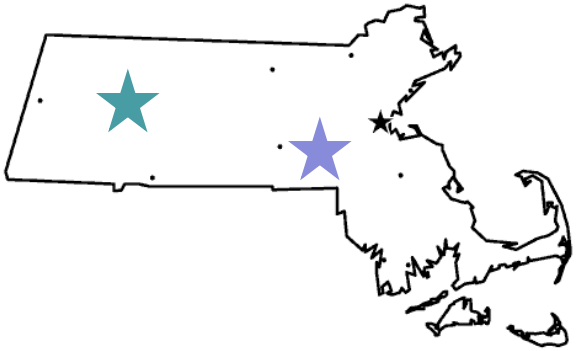
wind?

weeds?

crop rotation?

...other?





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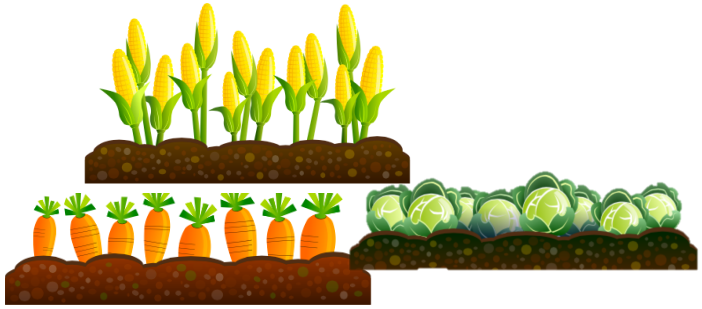

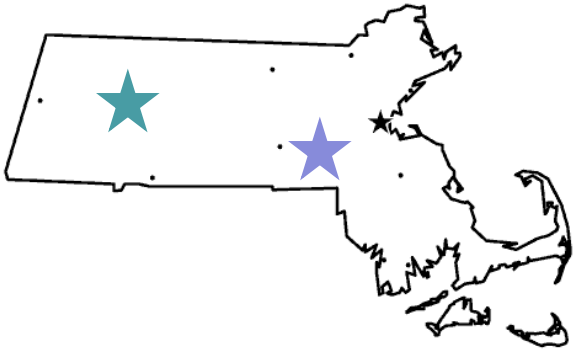

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Questions?

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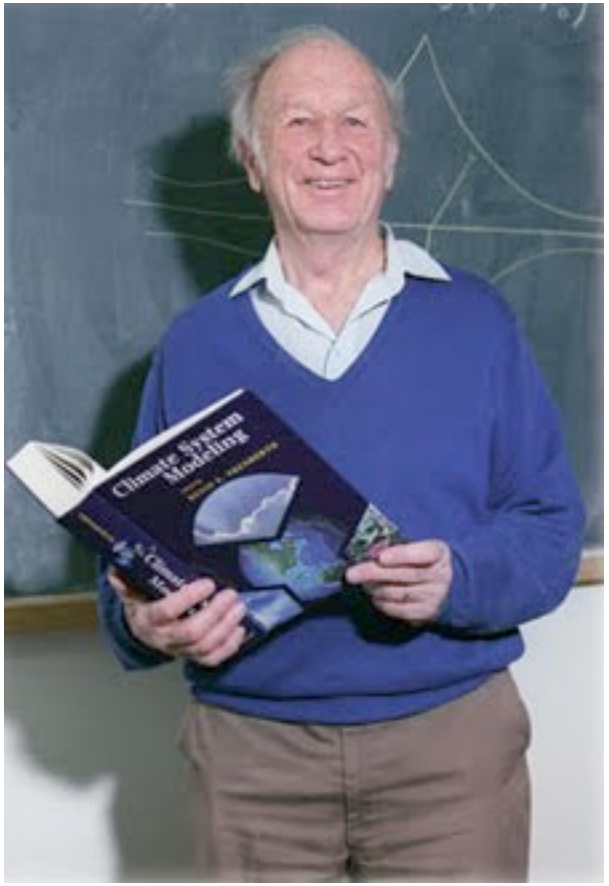
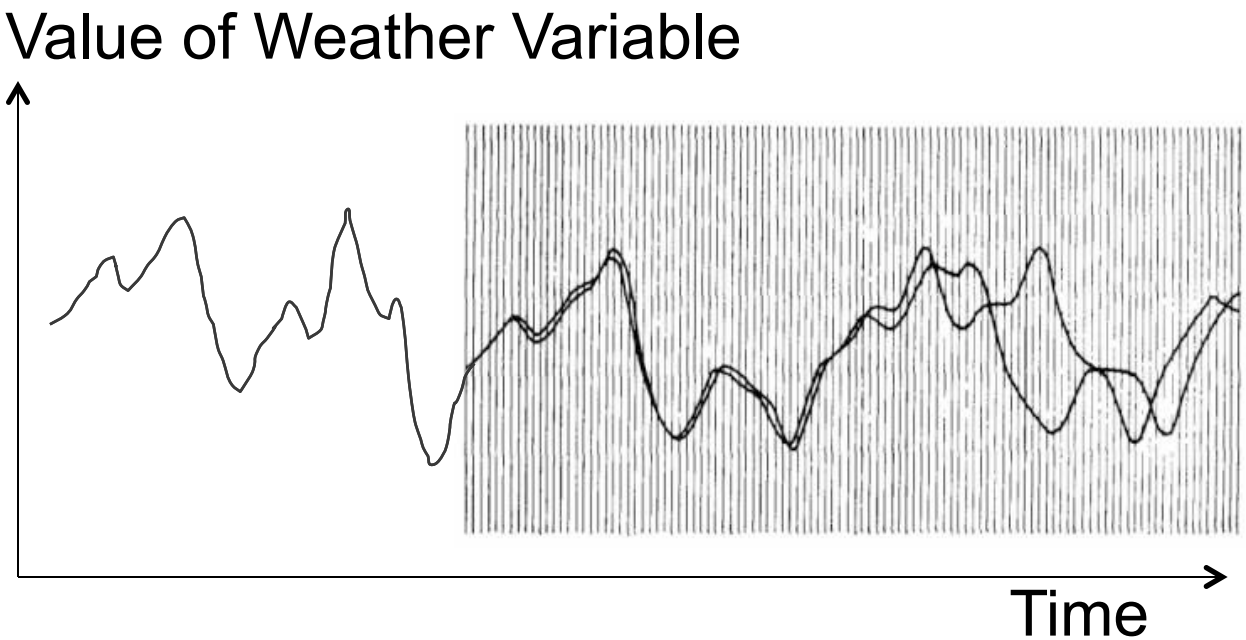
Ed Lorenz and the Discovery of Chaos, 1961



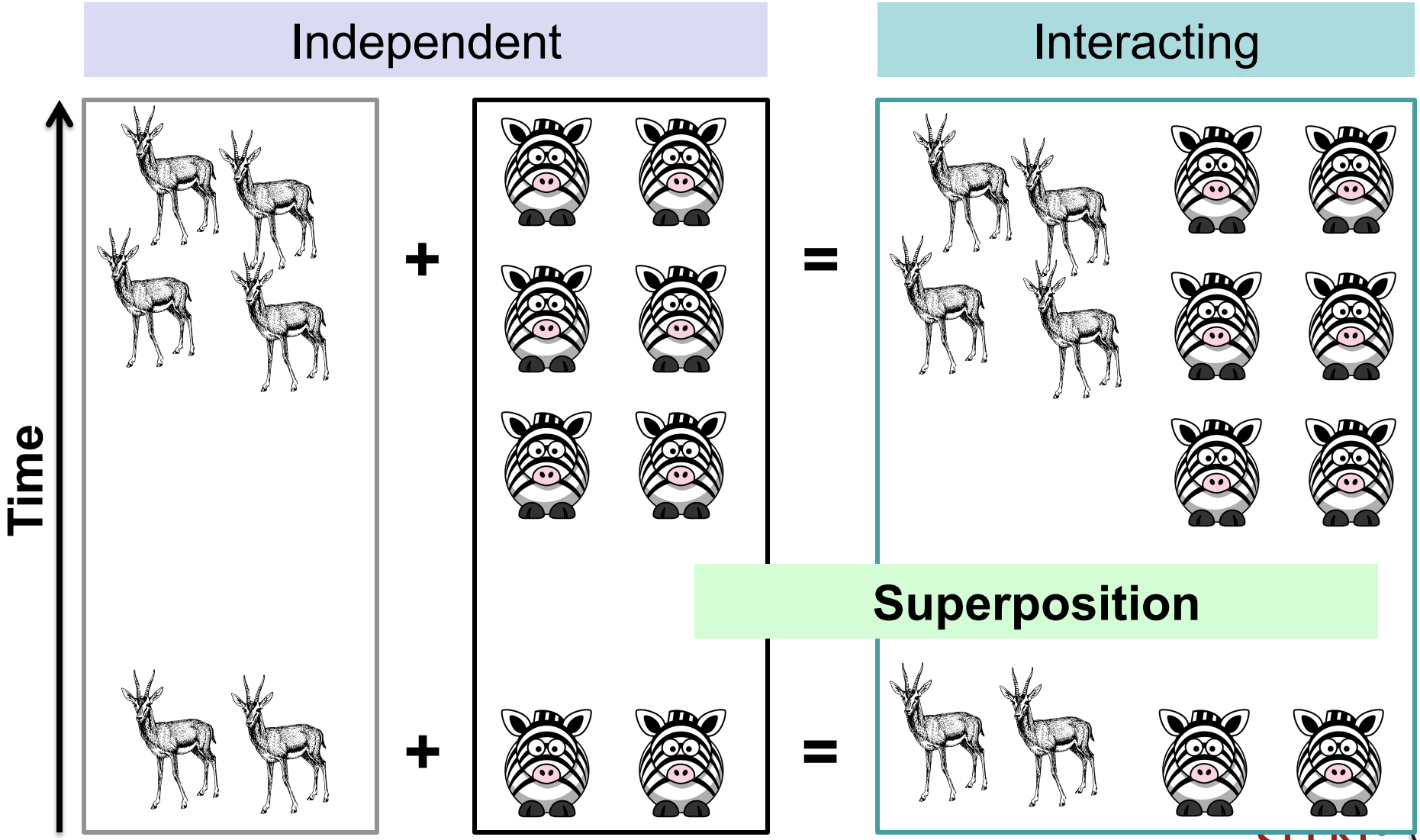


$$\begin{array}{r} 0.506127 \\ \text{vs} \quad \underline{0.506} \\ 0.000127 \end{array}$$

error of about 3 in 10,000!

http://www.ucar.edu/communications/quarterly/spring08/ed_lorenz.jsp
http://mcherm.com/permalinks/1/the-butterfly-effect

A Linear Model



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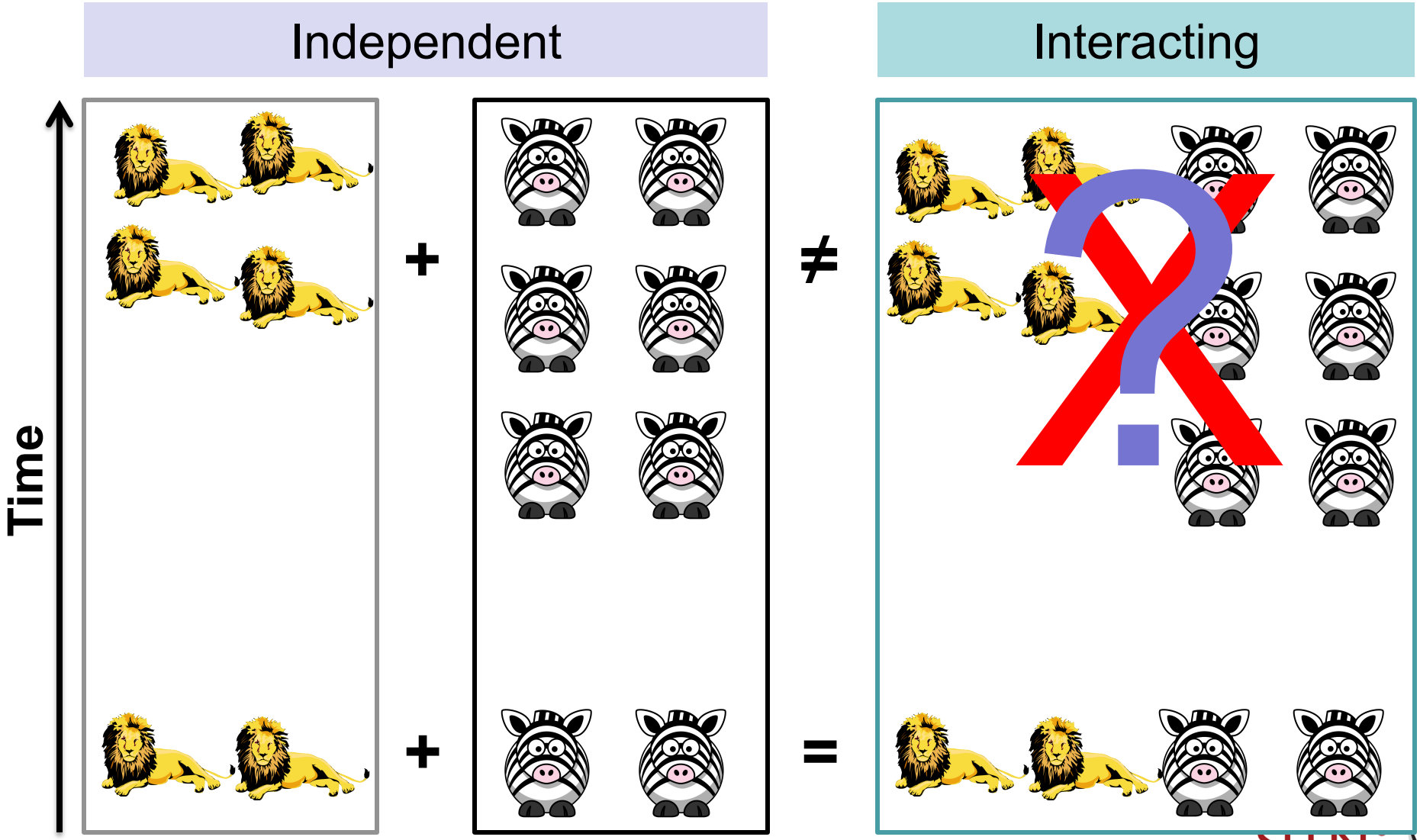
http://openclipart.org/detail/36937/gazelle-by-papapishu

A **Non**linear Model



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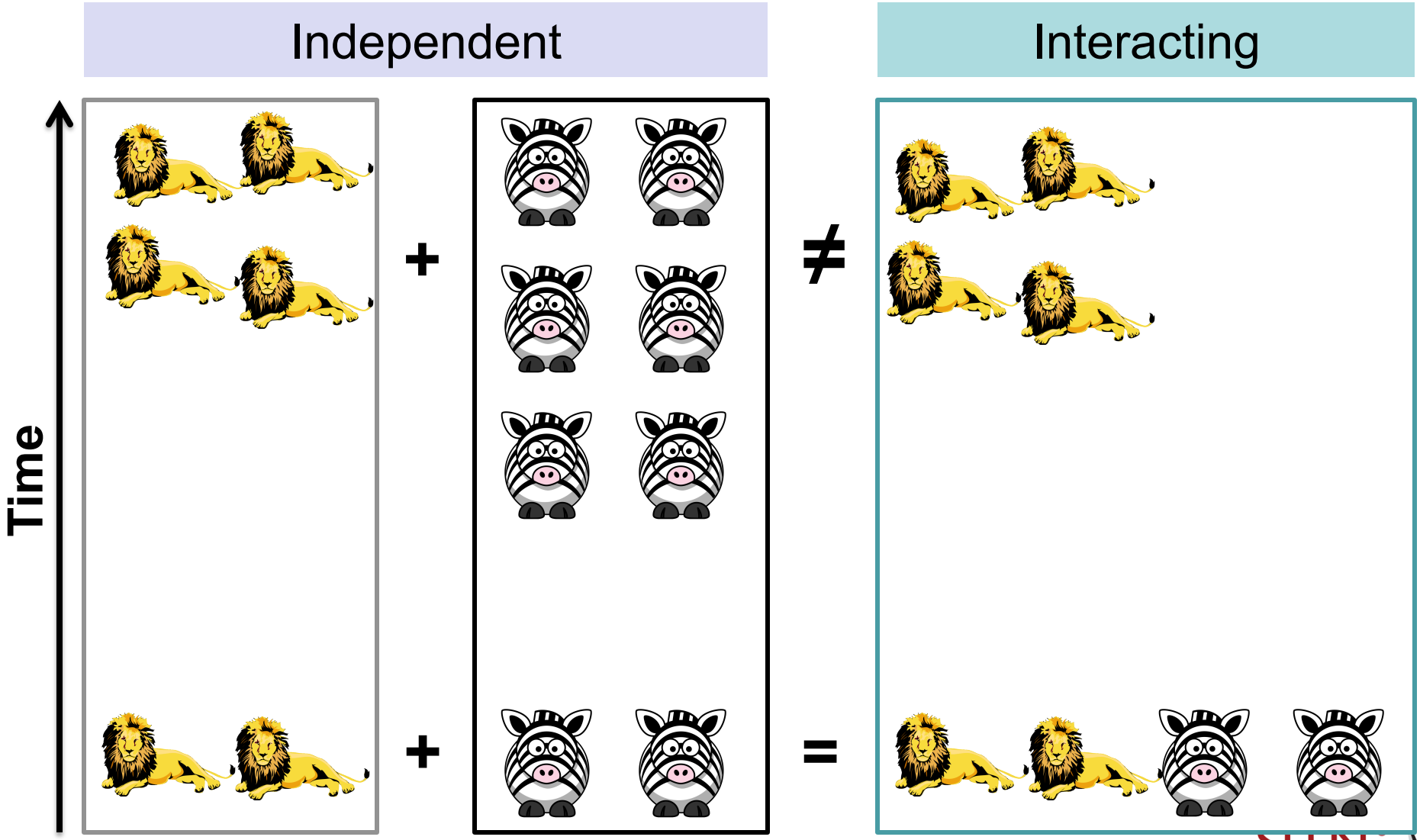
A **Non**linear Model



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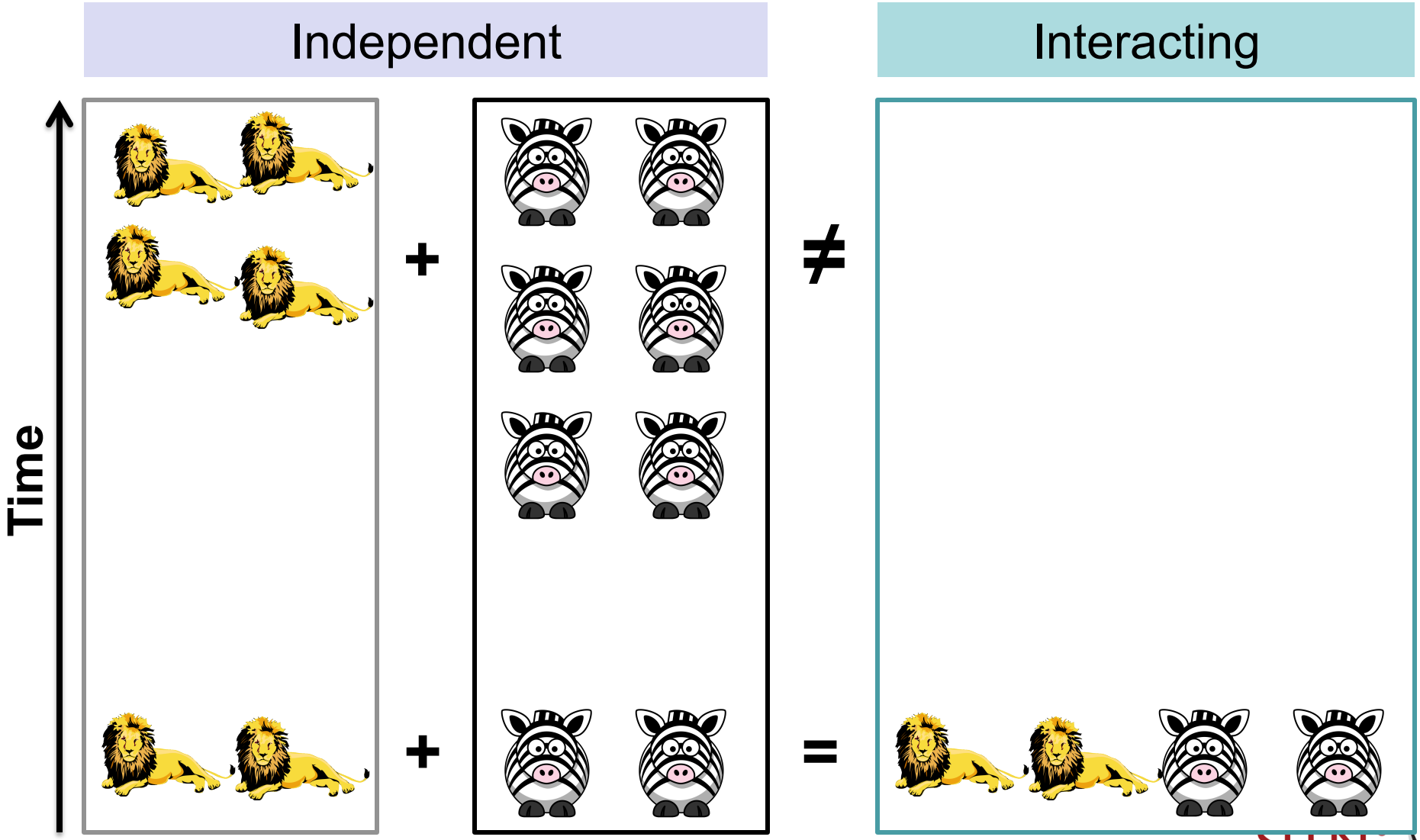

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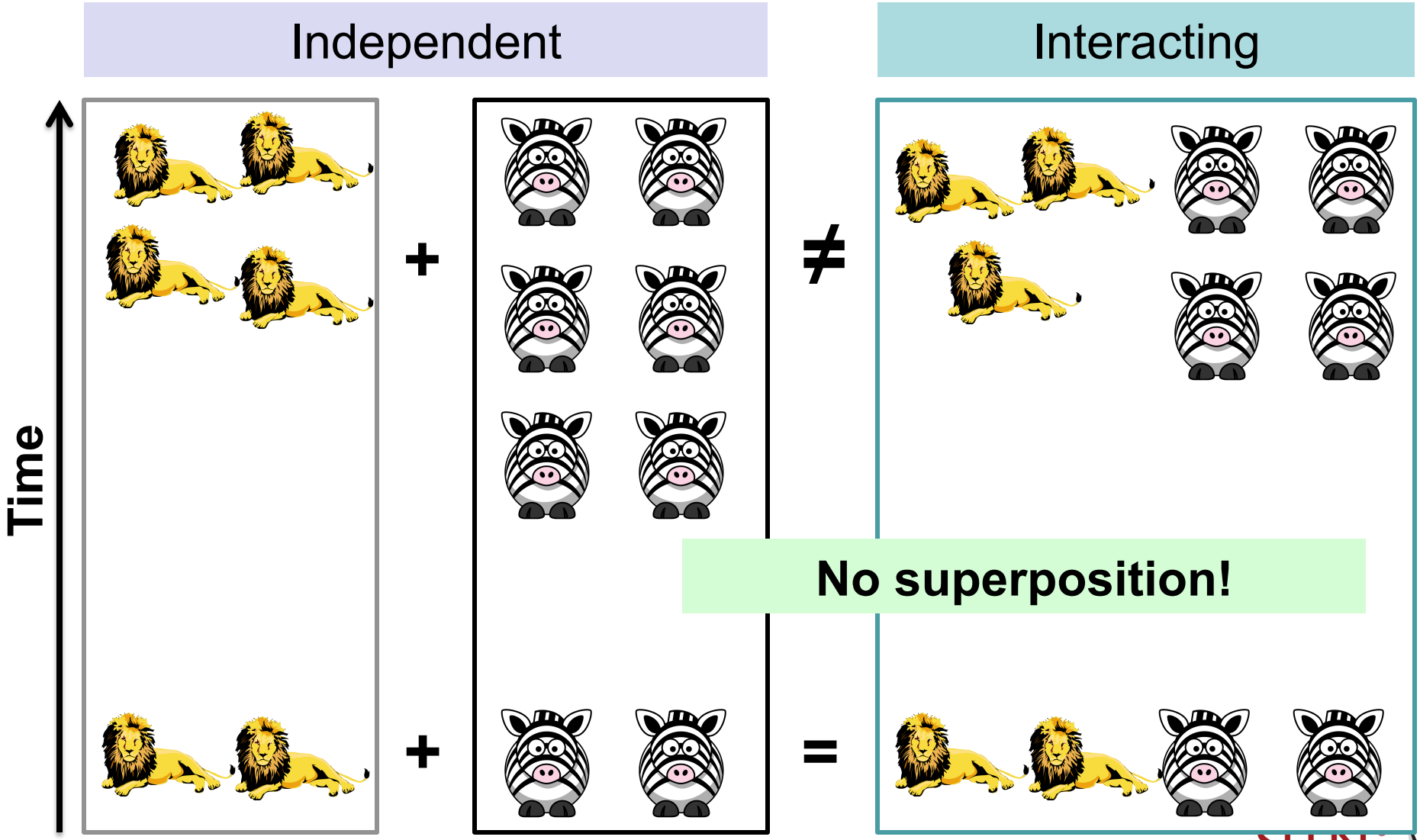

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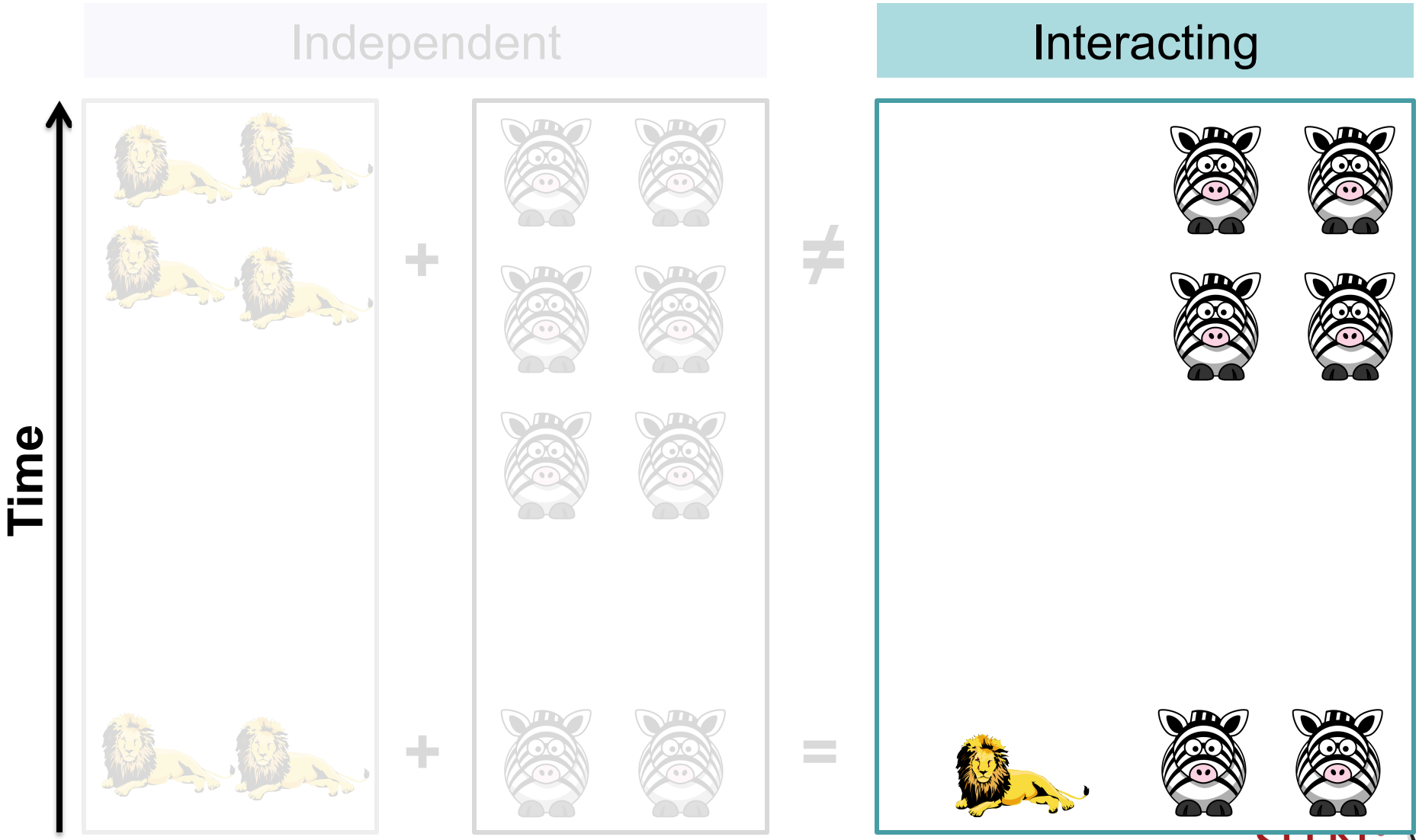

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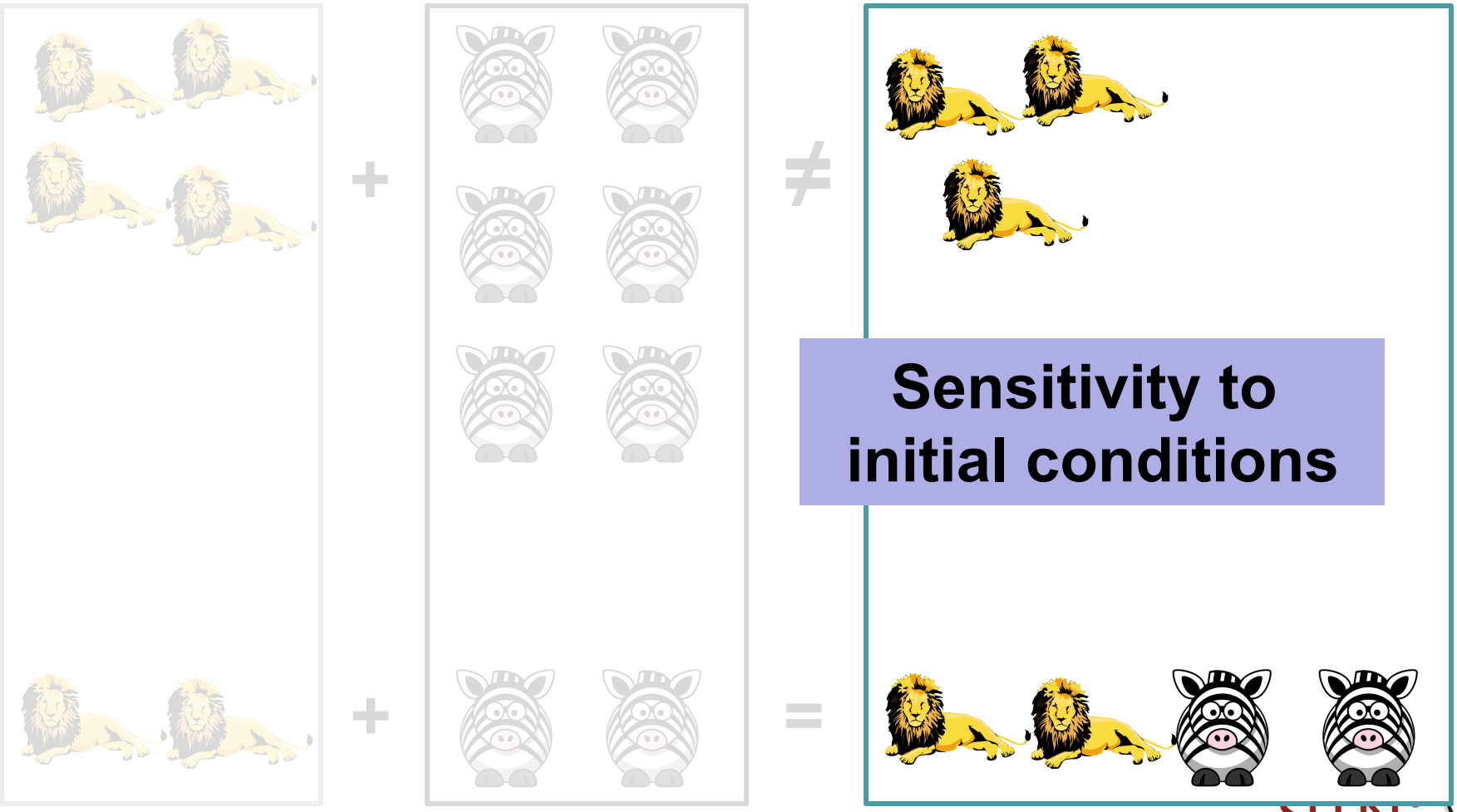
http://openclipart.org/detail/31975/architetto----leone-02-by-anonymous

A **Non**linear Model

Independent

Interacting

Time



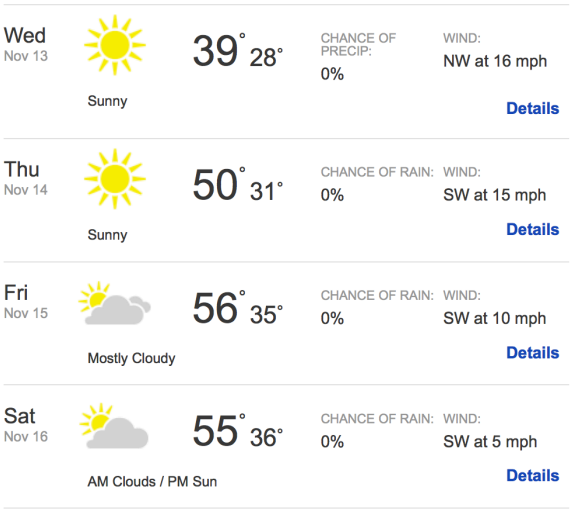
Sensitivity to initial conditions

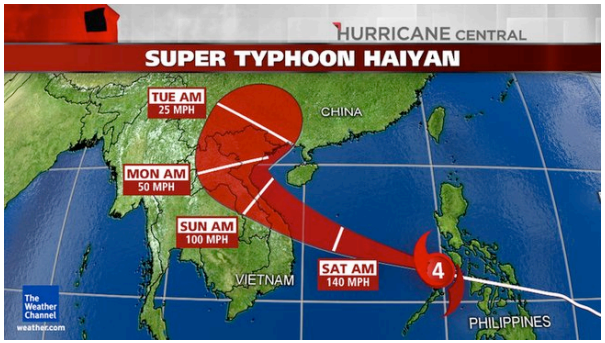
http://openclipart.org/detail/22287/cartoon-zebra-by-studiofibonacci
http://openclipart.org/detail/31975/architetto----leone-02-by-anonymous

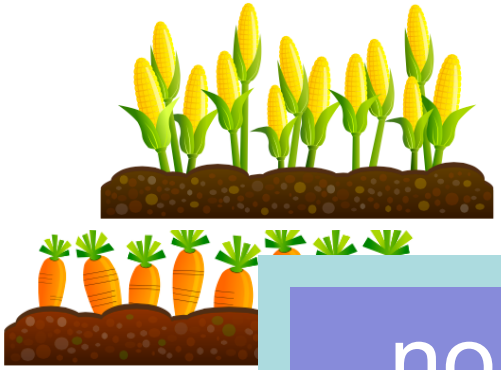
Summary: Linear vs Non-linear

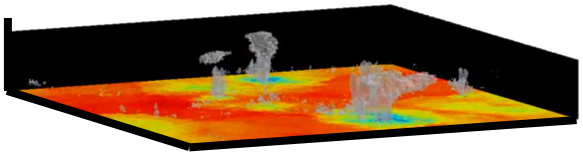
	Linear	Non-linear
Superposition – The result of the combination is the same as the combination of the separate results	**Yes**!	**Nope**

Modeling our world









...nonlinearity is everywhere!

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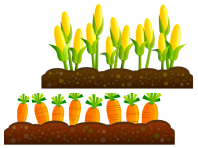
Summary



- Models help us study nature
- Nonlinear models
  - ~~Superposition~~
  - Sensitivity to initial conditions
  - A note about feedbacks and interactions
- Nonlinearity is everywhere!

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Part II

Butterfly effect and weather forecasting

Lei Zhu
Phd student
Atmospheric chemistry

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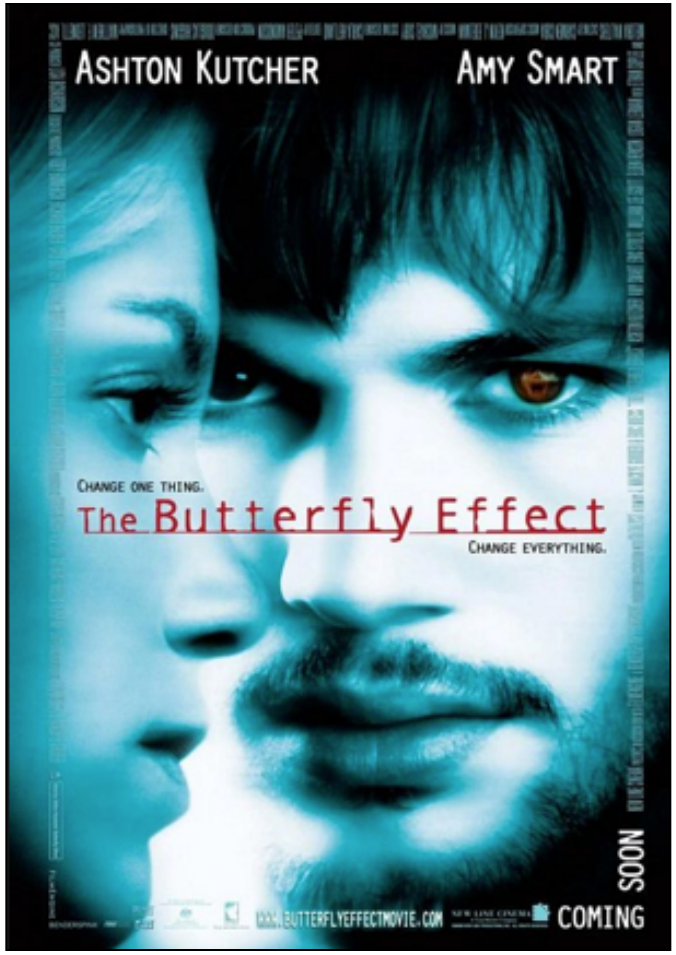
Butterfly effect in popular culture



When a butterfly flaps its wings in one part of the world it can cause a hurricane in another part of the world

http://www.manoneileen.com/2011/03/17/sci-fi-movie-evaluation-the-butterfly-effect/
http://www.technekes.com/blog/category/financial-services-practice

A metaphor from Edward Lorenz

*One meteorologist remarked that if the theory were correct, one flap of a **seagull's** wings would be enough to alter the course of the weather forever.*

-Lorenz, 1963

*Predictability: Does the Flap of a **Butterfly's** Wings in Brazil set off a Tornado in Texas?*

-Lorenz, 1972

Seagull evolved into the more poetic butterfly

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“Nail effect”: An old poem

http://scottgustafson.com/WN_books_NR7.html

Lessons we learn:

A tiny change can lead to a huge difference in the final results

For want of a nail the shoe was lost,
for want of a shoe the horse was lost;
and for want of a horse the rider was lost;
being overtaken and slain by the enemy,
all for want of care about a horse-shoe nail.

-Benjamin Franklin

Butterfly effect: from the perspective of chaos theory

- Sensitivity to the initial conditions
- Every observation has errors
- The error will be propagated through the nonlinear system
- Thus, it's almost impossible to make long time predictions in a nonlinear system

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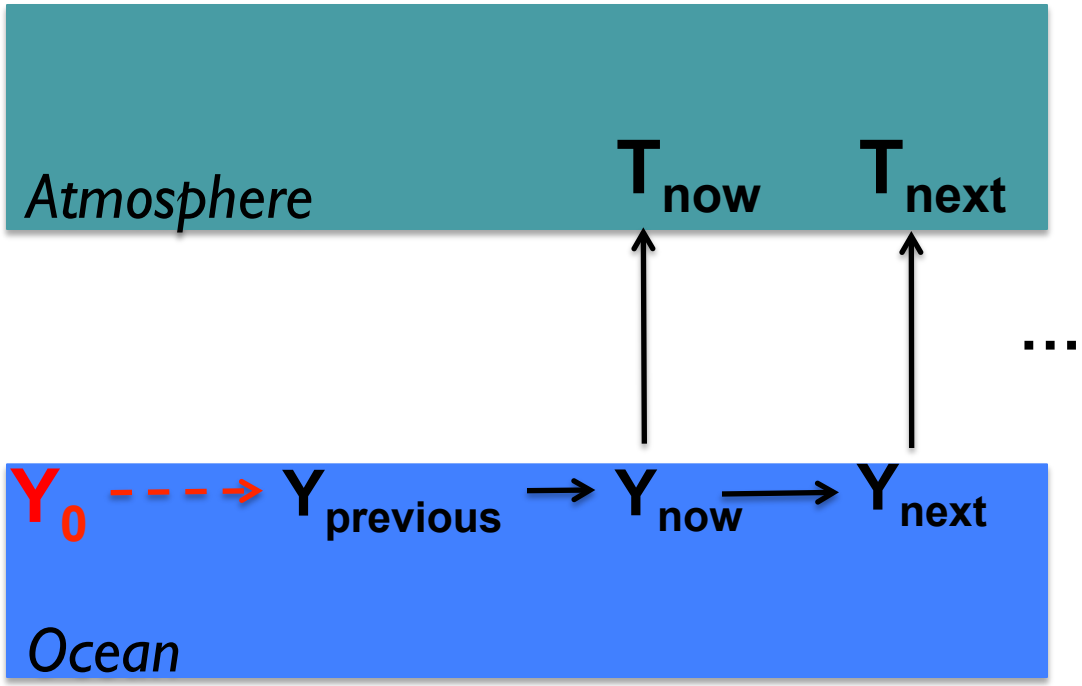
Butterfly effect: An example

Consider a temperature forecasting model:

$$\begin{cases} Y_t = 1.5 \times {Y_{t-1}}^2 - 1 \\ T_t = 30 \times (Y_t + 1) + 60 \end{cases}$$



Initial condition from observation

Butterfly effect: Any pattern?

Can you see any pattern in simulations with different initial conditions?

Consider our temperature model again:

Weather forecasting

- Short term forecasting (~12h) is very accurate
- Long term forecasting is impossible.
- But we can predict the probability!

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Butterfly effect: Predict the probability

Run the simulation for 1000 times with slightly different initial conditions.

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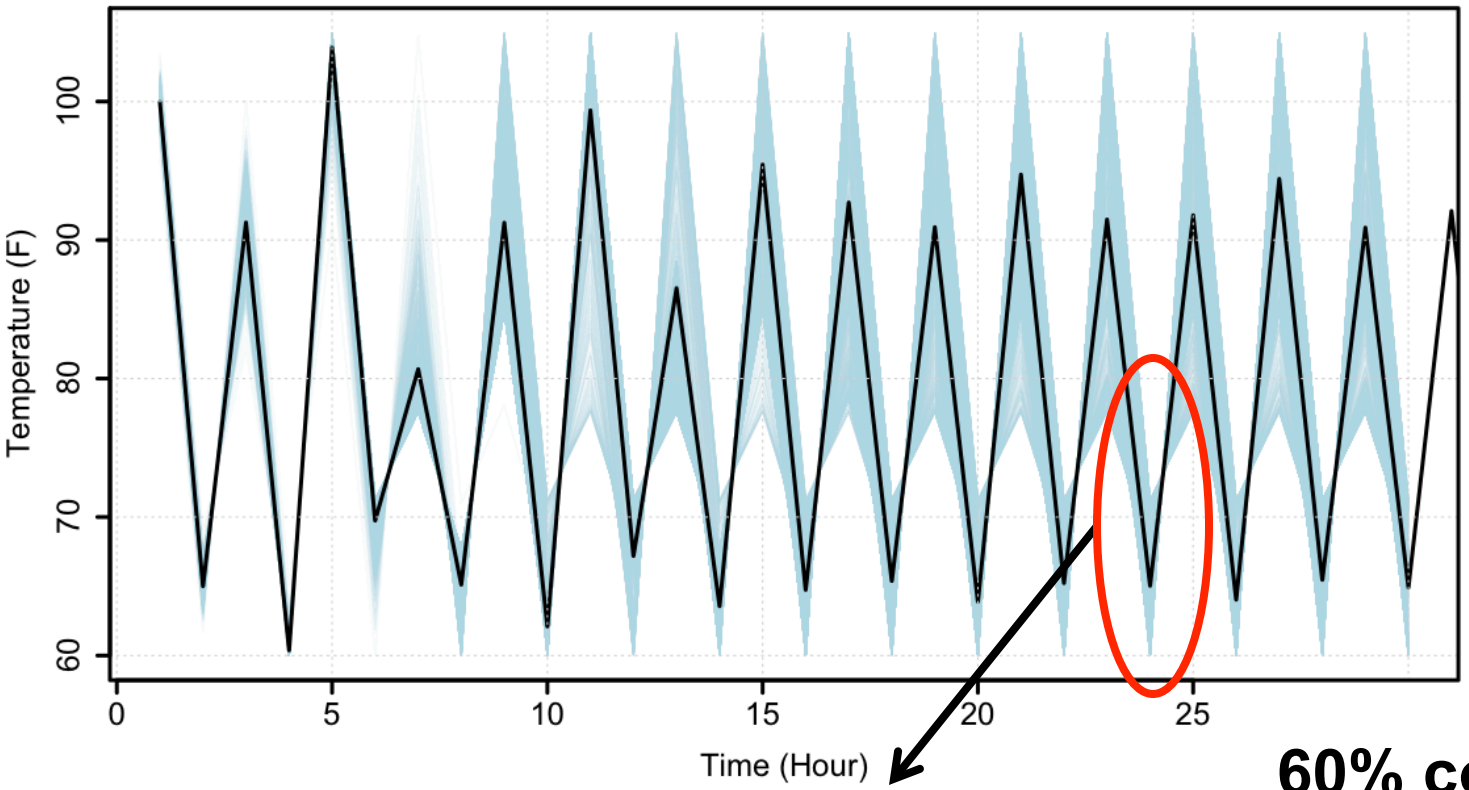
Butterfly effect: Predict the probability

Time series for 1000 simulations



60% confidence

600 of the 1000 simulations:
Range from 60.9-68.3 F

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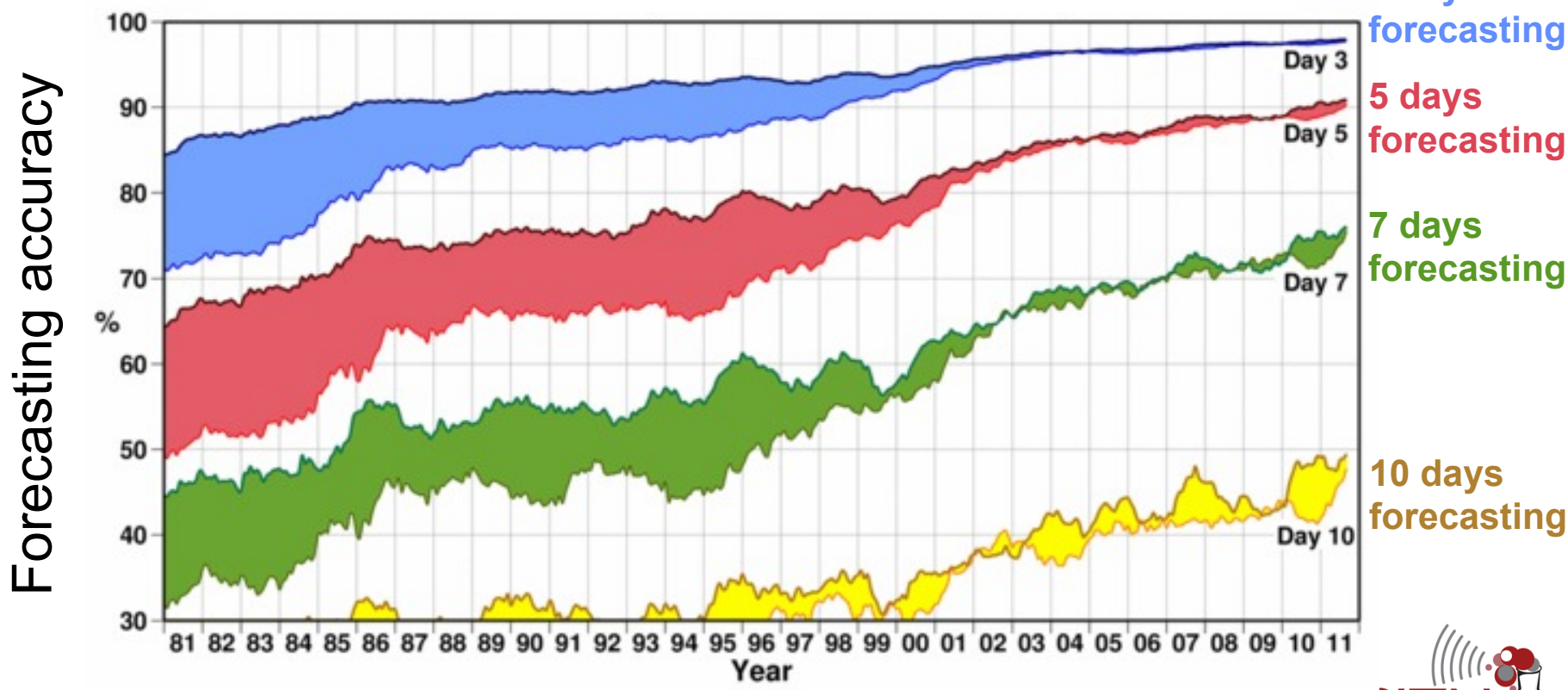
Weather forecasting

- What improves our prediction capabilities?

Forecasting accuracy from 1981 to 2011



Summary

- Butterfly effect is nothing more than a metaphor
- Long term weather forecasting is impossible
- But we can predict the probability
- There is a general pattern/rule/structure in a chaotic system

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Part III:
Structure in Chaos

Stephen Portillo

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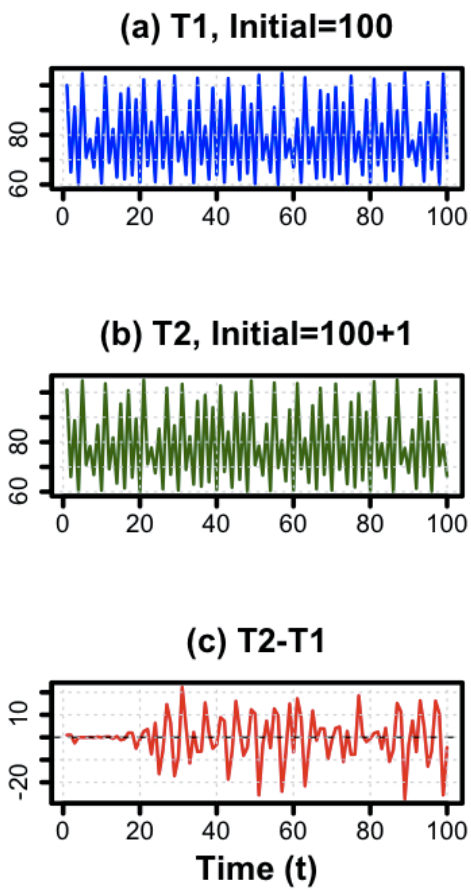
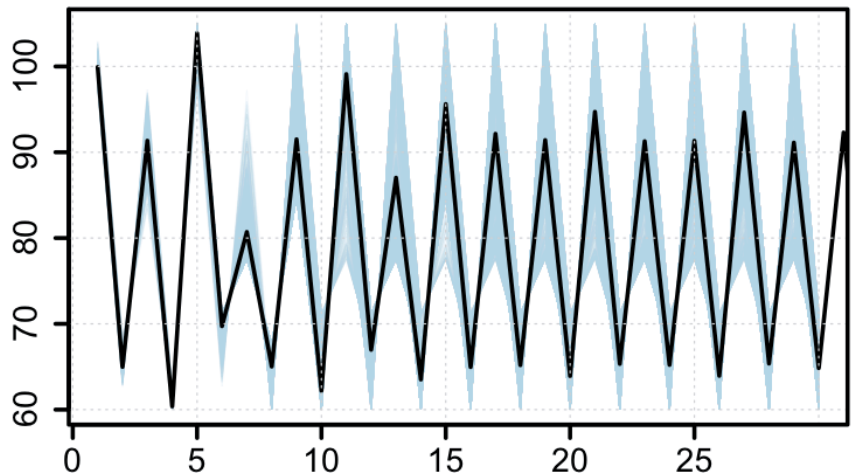
Understanding the Patterns

It's impossible to predict the exact behaviour of non-linear systems,



But non-linear systems have patterns.



What can we learn about these patterns?

Outline

- The Logistic Equation
- Non-Linear Math and Self-Similarity
- Self-Similarity in Nature

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AN EXAMPLE:
THE LOGISTIC EQUATION

Simplify the Math

To <u>predict</u>, we need a complicated model

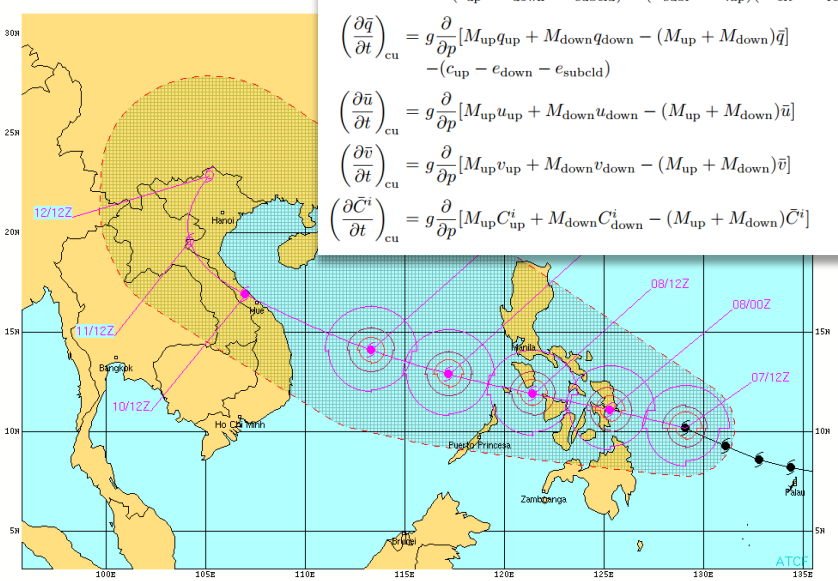
$$\left(\frac{\partial \bar{s}}{\partial t}\right)_{\mathrm{cu}} = g\frac{\partial}{\partial p}[M_{\mathrm{up}}s_{\mathrm{up}} + M_{\mathrm{down}}s_{\mathrm{down}} - (M_{\mathrm{up}} + M_{\mathrm{down}})\bar{s}] + L(c_{\mathrm{up}} - e_{\mathrm{down}} - e_{\mathrm{subcld}}) - (L_{\mathrm{subl}} - L_{\mathrm{vap}})(M_{\mathrm{elt}} - F_{\mathrm{rez}})$$

$$\left(\frac{\partial \bar{q}}{\partial t}\right)_{\mathrm{cu}} = g\frac{\partial}{\partial p}[M_{\mathrm{up}}q_{\mathrm{up}} + M_{\mathrm{down}}q_{\mathrm{down}} - (M_{\mathrm{up}} + M_{\mathrm{down}})\bar{q}] - (c_{\mathrm{up}} - e_{\mathrm{down}} - e_{\mathrm{subcld}})$$

$$\left(\frac{\partial \bar{u}}{\partial t}\right)_{\mathrm{cu}} = g\frac{\partial}{\partial p}[M_{\mathrm{up}}u_{\mathrm{up}} + M_{\mathrm{down}}u_{\mathrm{down}} - (M_{\mathrm{up}} + M_{\mathrm{down}})\bar{u}]$$

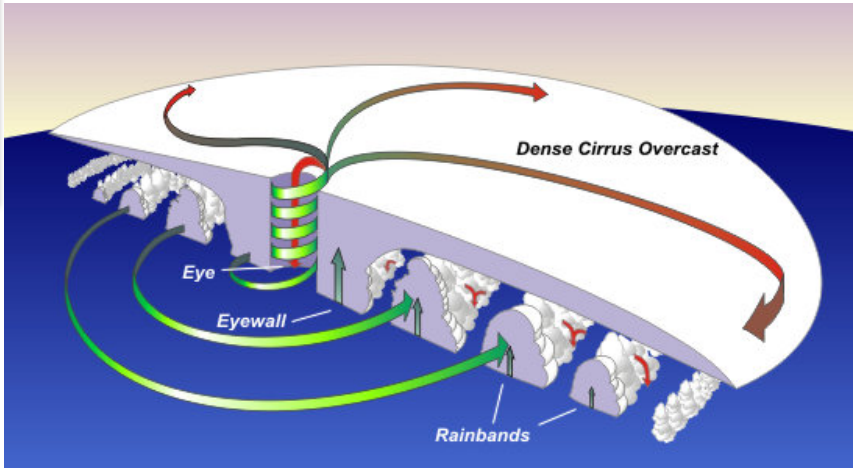
$$\left(\frac{\partial \bar{v}}{\partial t}\right)_{\mathrm{cu}} = g\frac{\partial}{\partial p}[M_{\mathrm{up}}v_{\mathrm{up}} + M_{\mathrm{down}}v_{\mathrm{down}} - (M_{\mathrm{up}} + M_{\mathrm{down}})\bar{v}]$$

$$\left(\frac{\partial \bar{C}^i}{\partial t}\right)_{\mathrm{cu}} = g\frac{\partial}{\partial p}[M_{\mathrm{up}}C^i_{\mathrm{up}} + M_{\mathrm{down}}C^i_{\mathrm{down}} - (M_{\mathrm{up}} + M_{\mathrm{down}})\bar{C}^i]$$



Joint Typhoon Warning Center
www.usno.navy.mil/JTWC/
European Centre for Medium-Range Weather Forecasts
www.ecmwf.int

To <u>understand</u>, we start with a simple model



National Weather Service
http://www.srh.noaa.gov/jetstream/tropics/tc_structure.htm

A First Ecological Model

- Consider an animal population that grows each year

$P_{next} = rP$

- P = population this year
- P_{next} = population next year
- r = reproductive rate

$r=2$

Year 1
- Population 100

Year 2
- Population 200

Year 3
- Population 400

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A First Ecological Model

- Consider an animal population that grows each year

$P\downarrow next = rP$

- P=population this year
- $P\downarrow next$ = population next year
- r=reproductive rate

$r=2$

Year 1 • Population 100	Year 1 • Population 200
Year 2 • Population 200	Year 2 • Population 400
Year 3 • Population 400	Year 3 • Population 800

Linear!

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The Logistic Equation

- Take the logistic equation

$$P_{next} = rP(1 - P/C)$$

- $C = carrying\ capacity$, the population where all resources would be exhausted

$r=2,\ C=1000$

Year 1
- Population 100

Year 2
- Population 180

Year 3
- Population 295

The Logistic Equation

- Take the logistic equation

$$P_{next} = rP(1 - P/C)$$

- C = *carrying capacity*, the population where all resources would be exhausted

$r=2,\ C=1000$

Year 1	Year 1
• Population 100	• Population 200
Year 2	**Year 2**
• Population 1[illegible]	• Population 320
Year 3	**Year 3**
• Population 295	• Population 435

Non-linear!

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Demo

- Carrying capacity of 1 000 000
- Reproductive rate of 2.5

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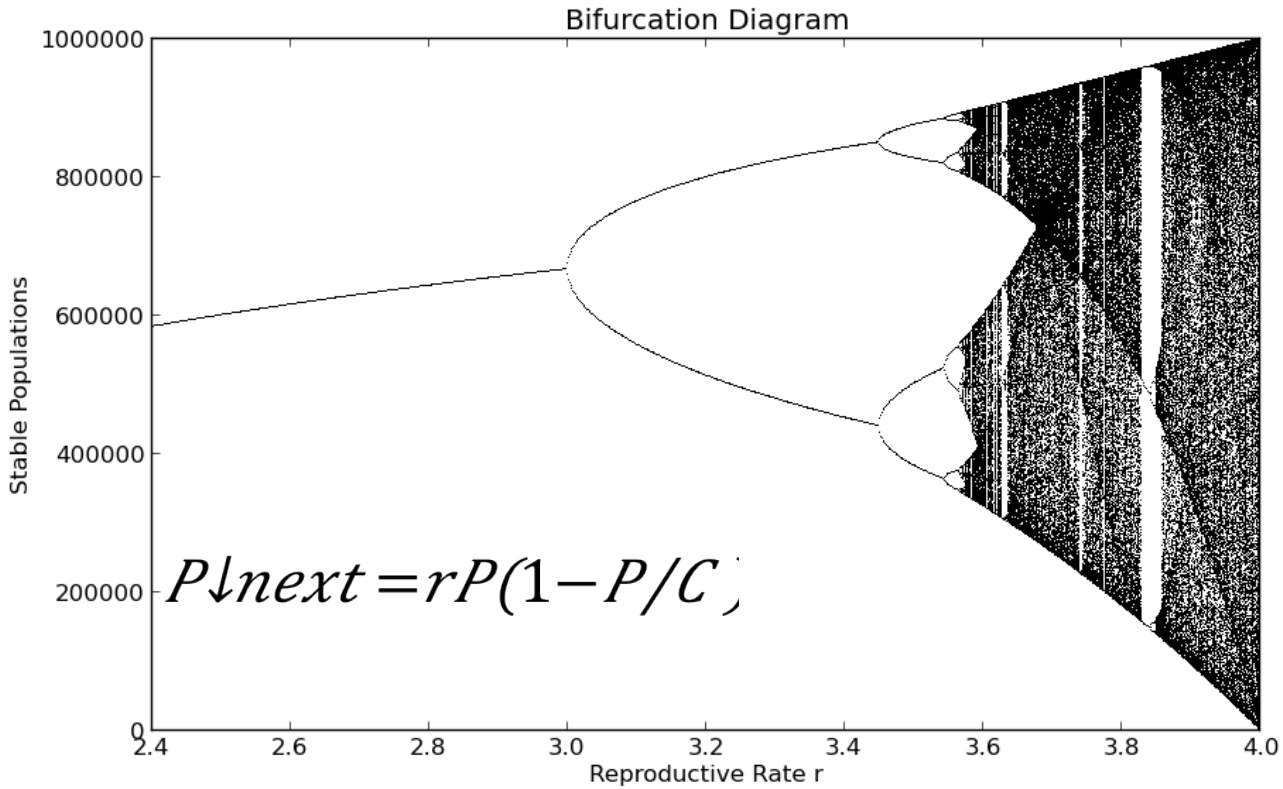
Meet the Bifurcation Diagram



Even a simple equation can give complex behaviour – if it's non-linear

Questions?

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NON-LINEAR MATH AND SELF-SIMILARITY

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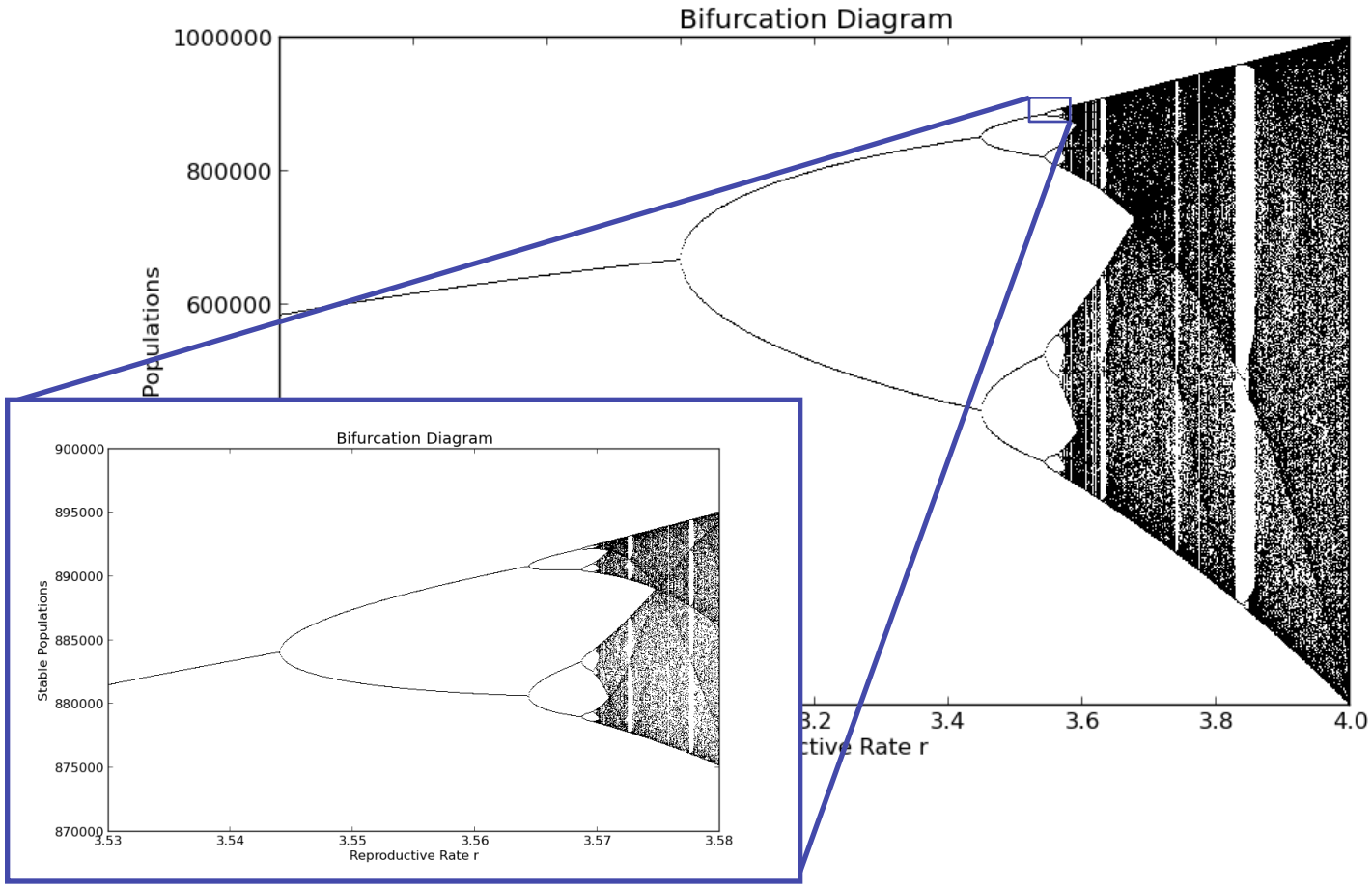
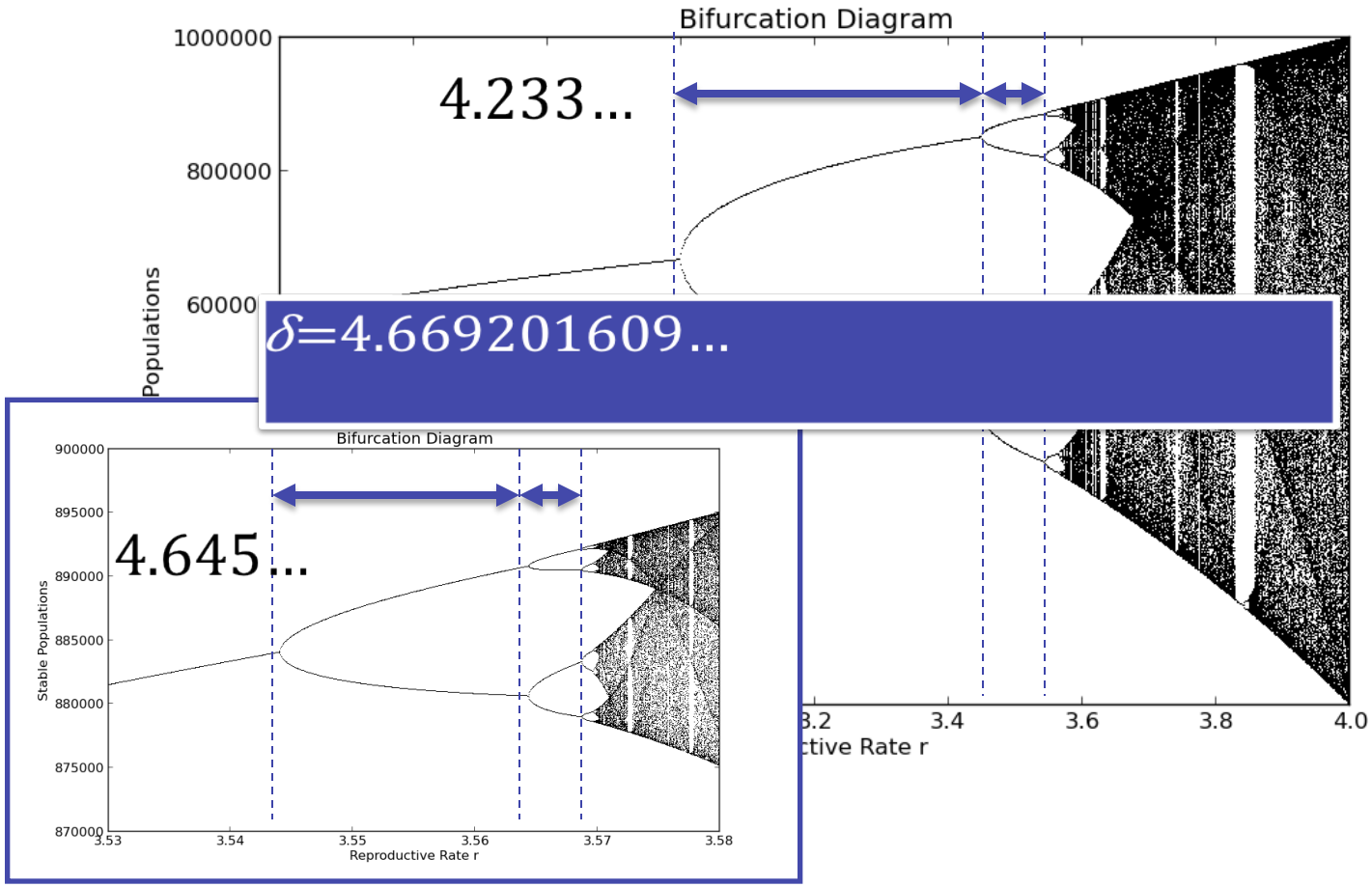
Self-Similarity



Feigenbaum's Constant



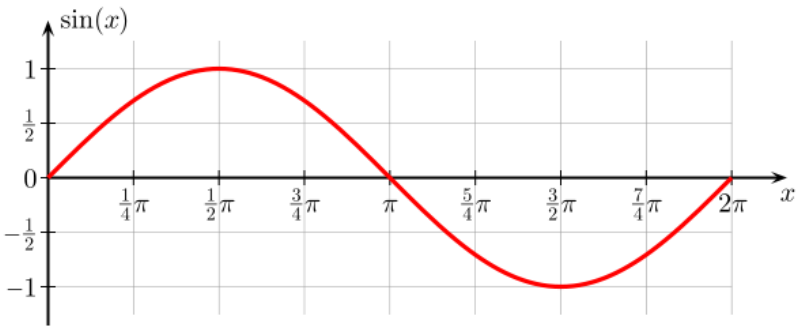
Universality

- Is this pattern just for the logistic equation?

$$P\downarrow next = rP(1-P/C)$$

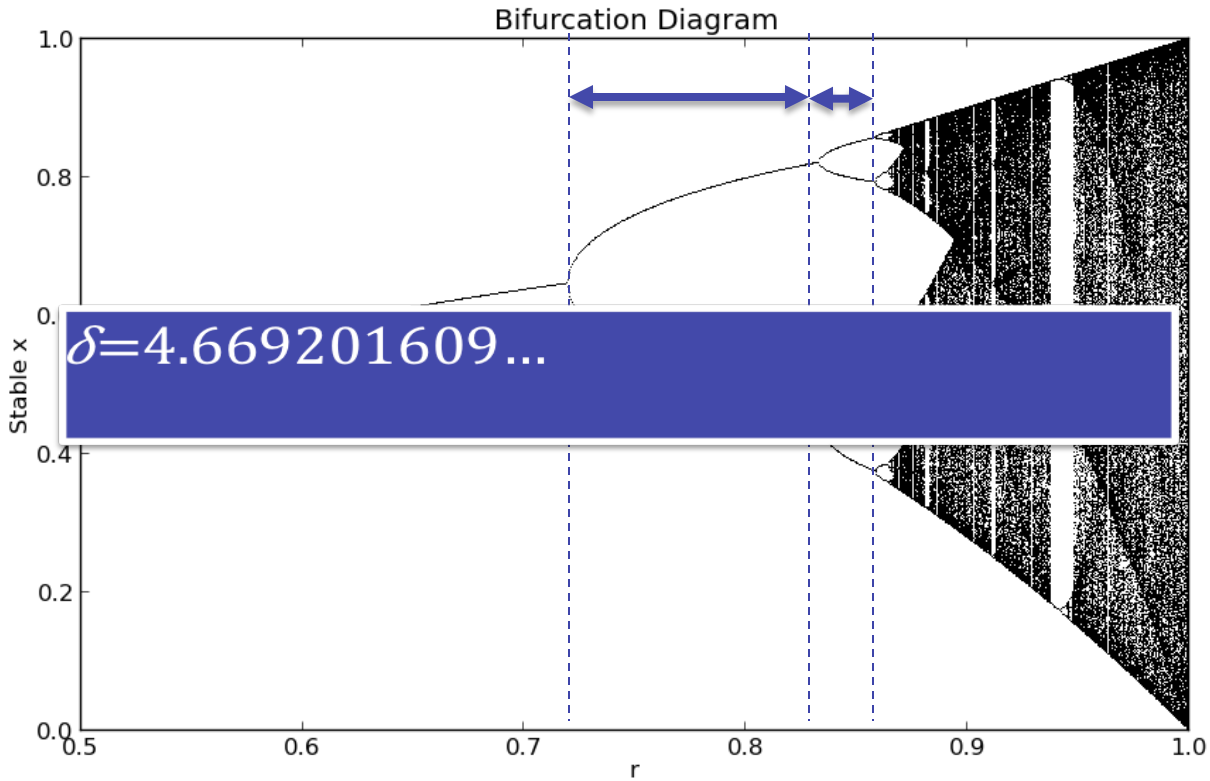
- Now for something completely different…

$$P\downarrow next = r\sin\pi P$$



Wikimedia Commons (user:Geek3)

Universality



This self-similarity arises from the math itself.

SELF-SIMILARITY IN NATURE

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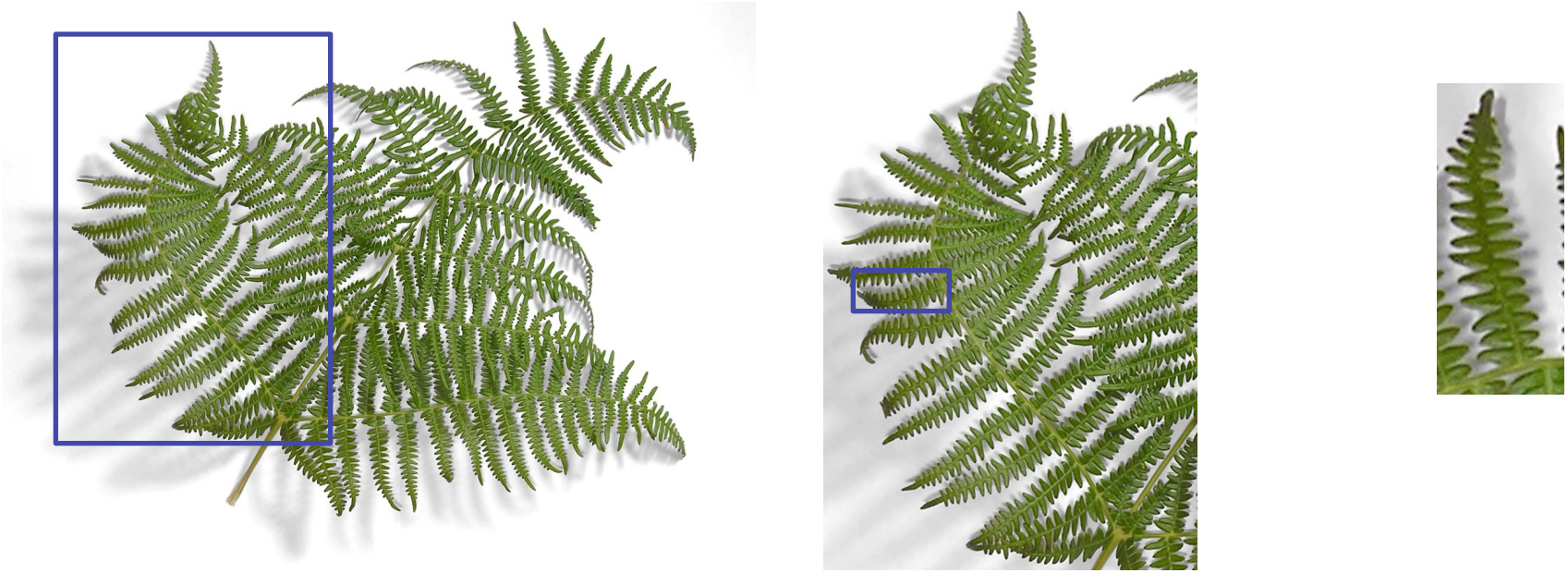
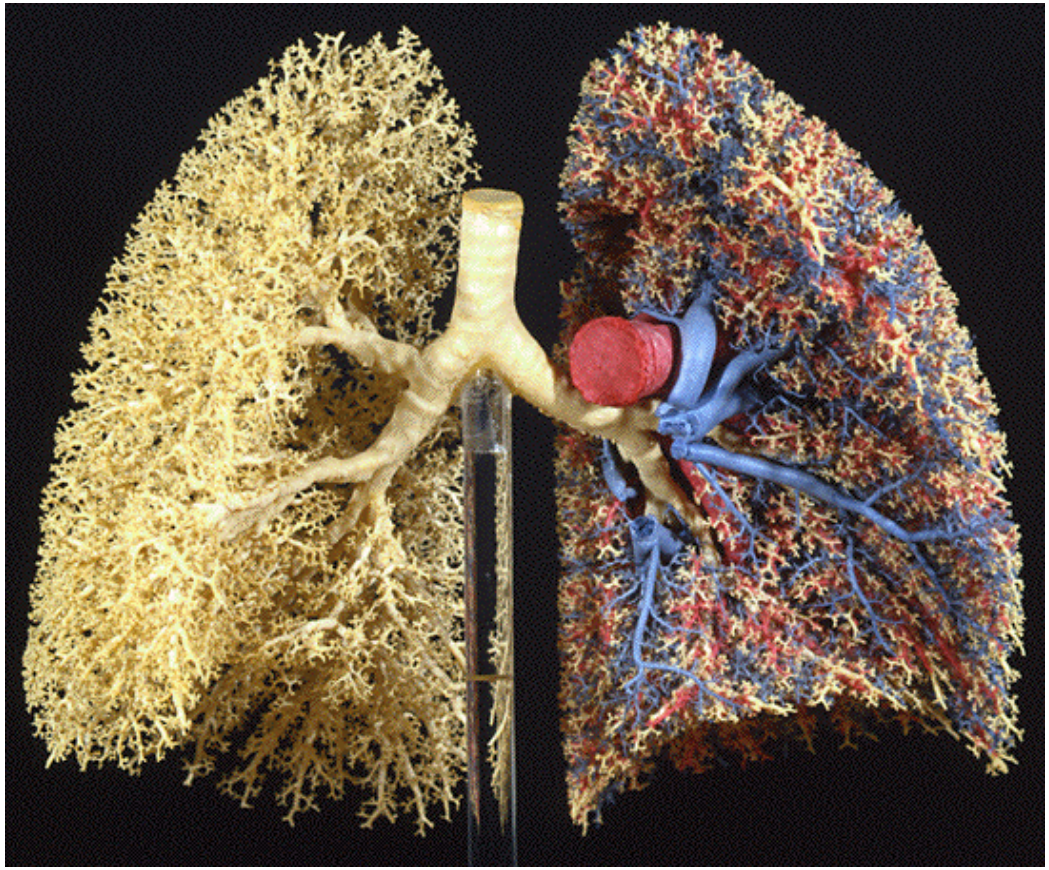
Self-Similarity in Nature



Wikimedia Commons (user:Olegiwit)

Self-Similarity in Nature



Fractal Geometry, Yale University
http://classes.yale.edu/fractals/

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Self-Similarity in Nature

Wikimedia Commons (user:Yann)

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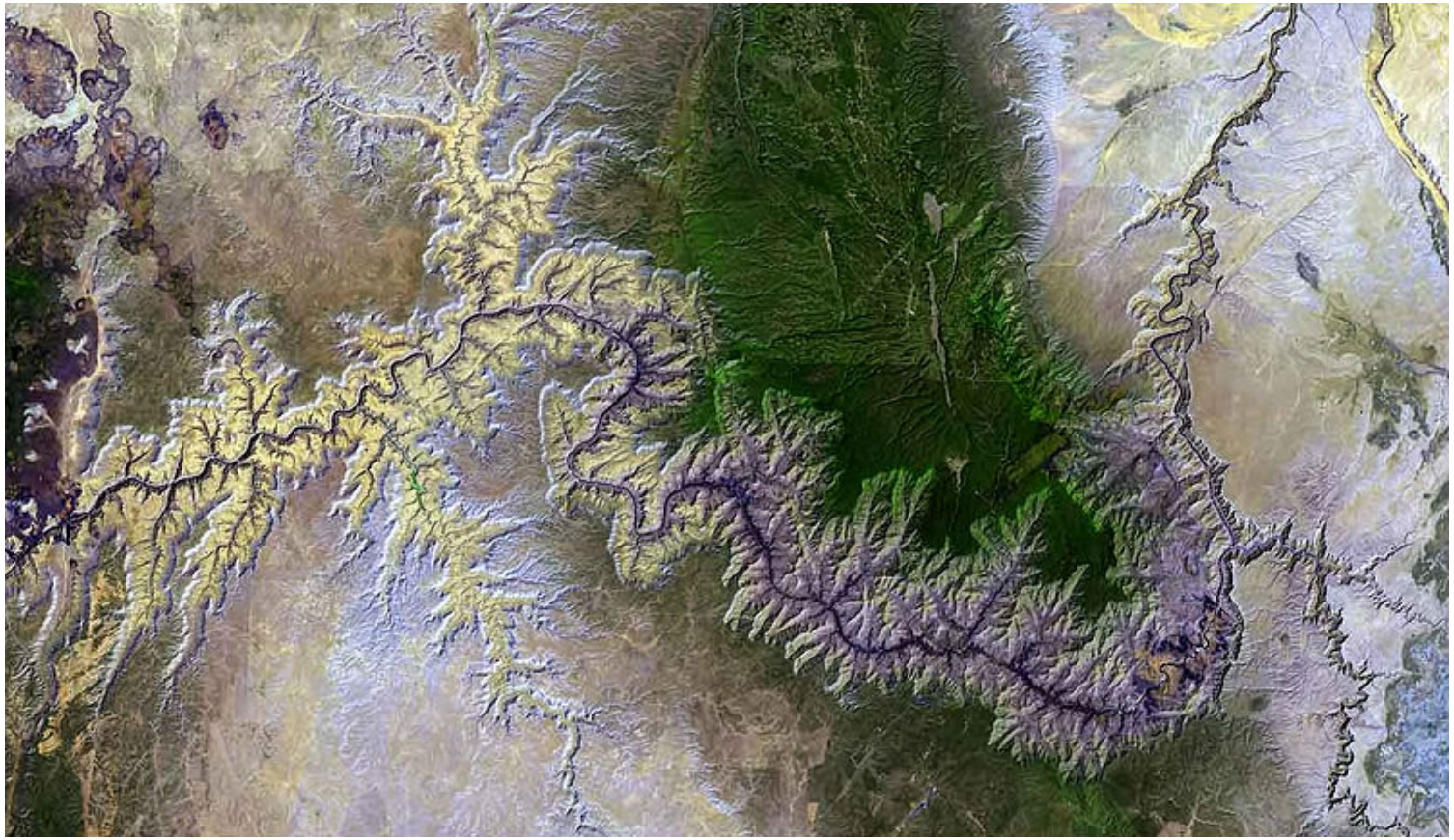
Self-Similarity in Nature



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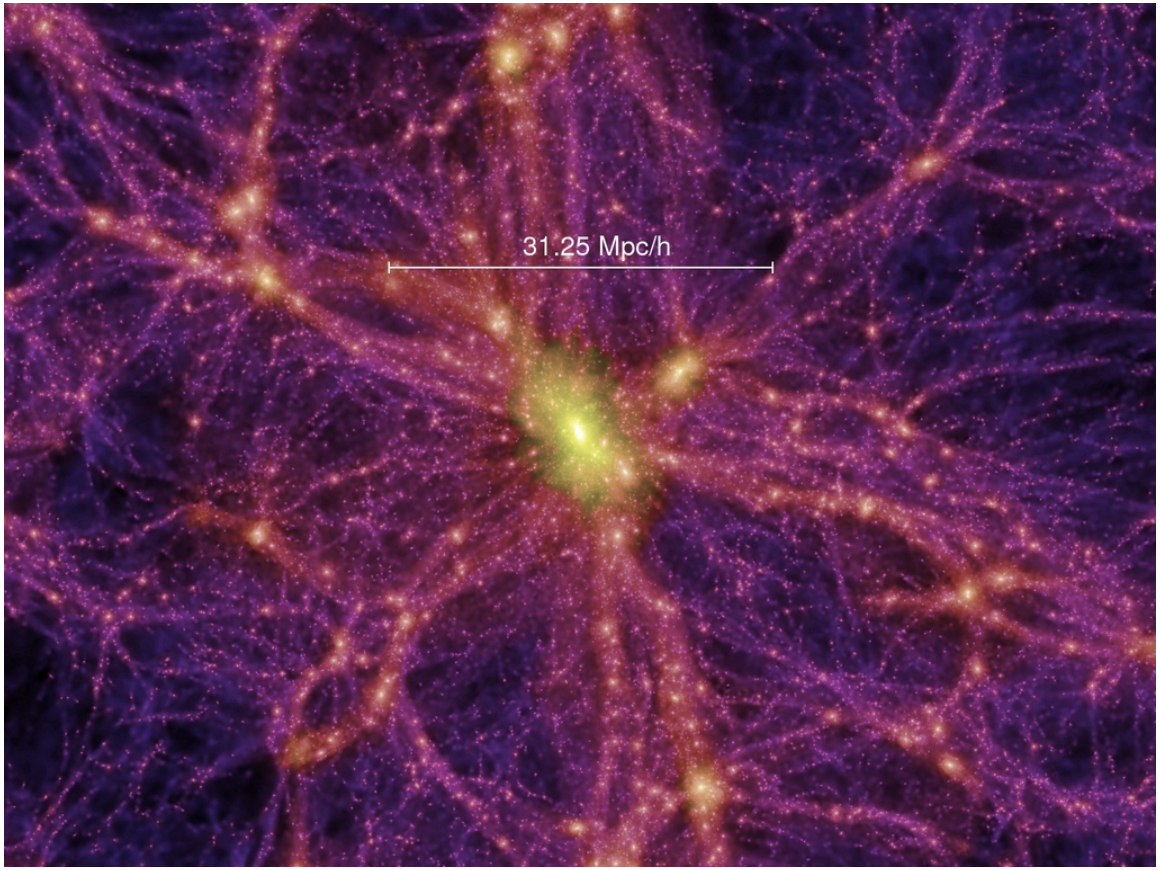
Self-Similarity in Nature

Smithsonian Institute

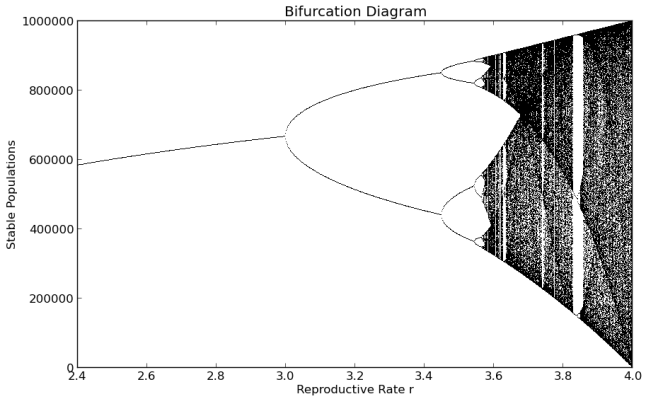
Self-Similarity in Nature

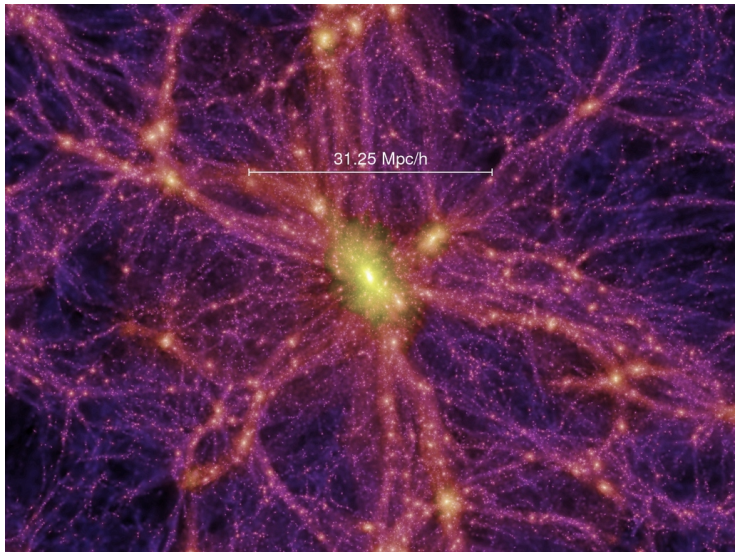


The Millenium Simulation
http://www.mpa-garching.mpg.de/galform/millennium/

The Nature of Self-Similarity

- Nature is described by non-linear math
- Non-linear math gives rise to self-similarity
- Nature has self-similarity arising from non-linear math





Conclusion

- We understand nature by using mathematical models, often non-linear
- We can't calculate the exact behaviour of non-linear models, but we can still make useful predictions
- Self-similarity arises from non-linear math itself, and is related to self-similarity in nature

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The Harvard Biomedical Graduate Students Organization (BGSO)

The Harvard/MIT COOP

Restaurant Associates

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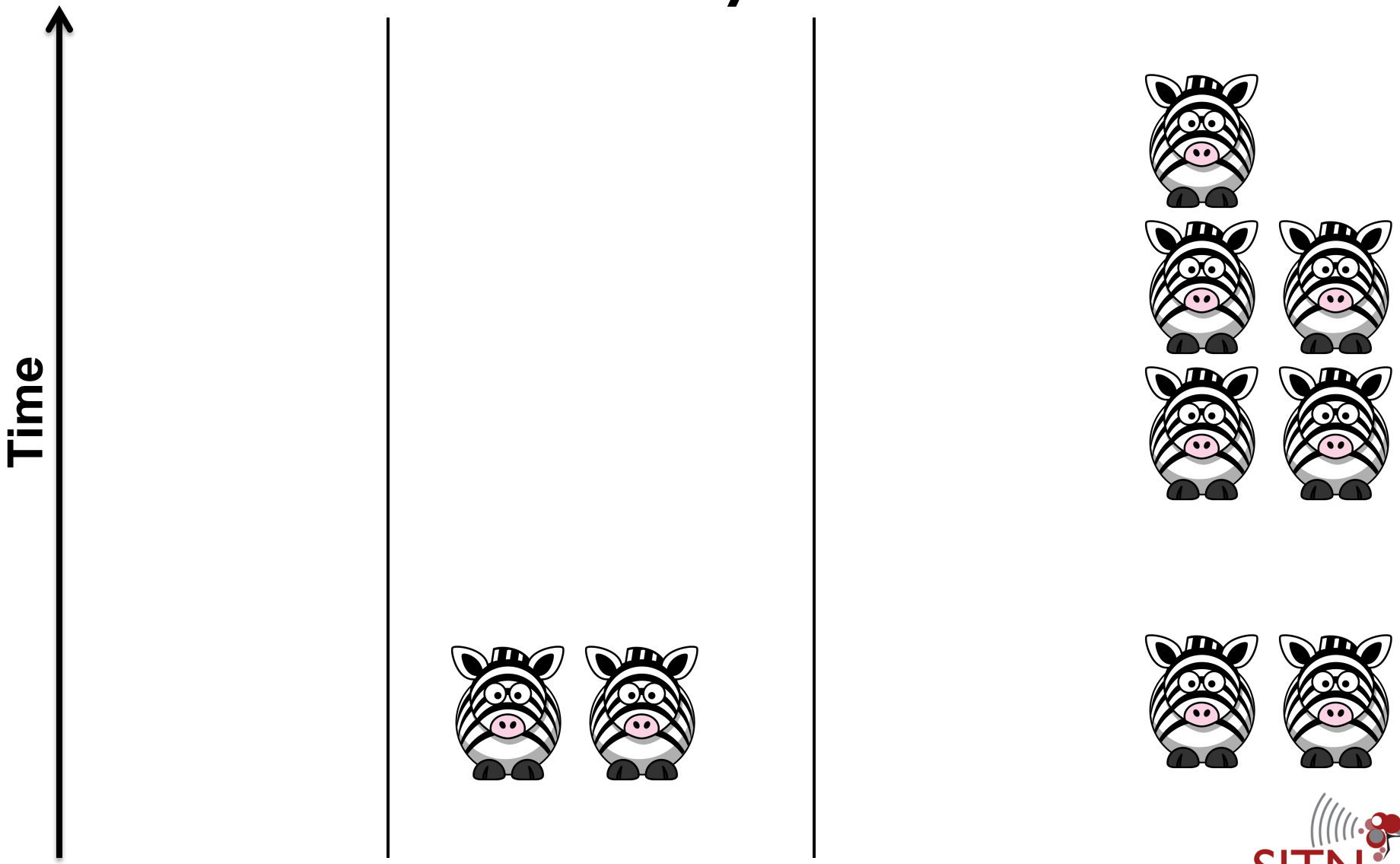
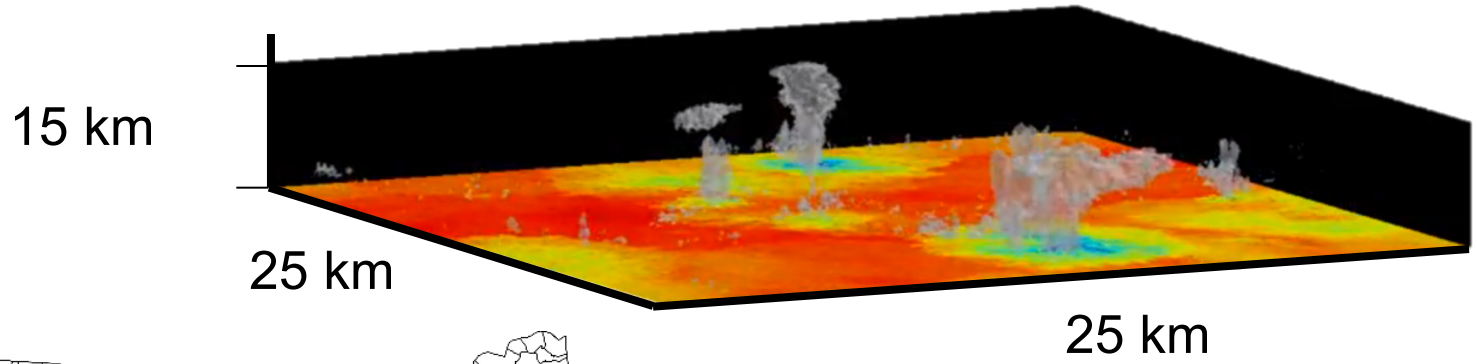
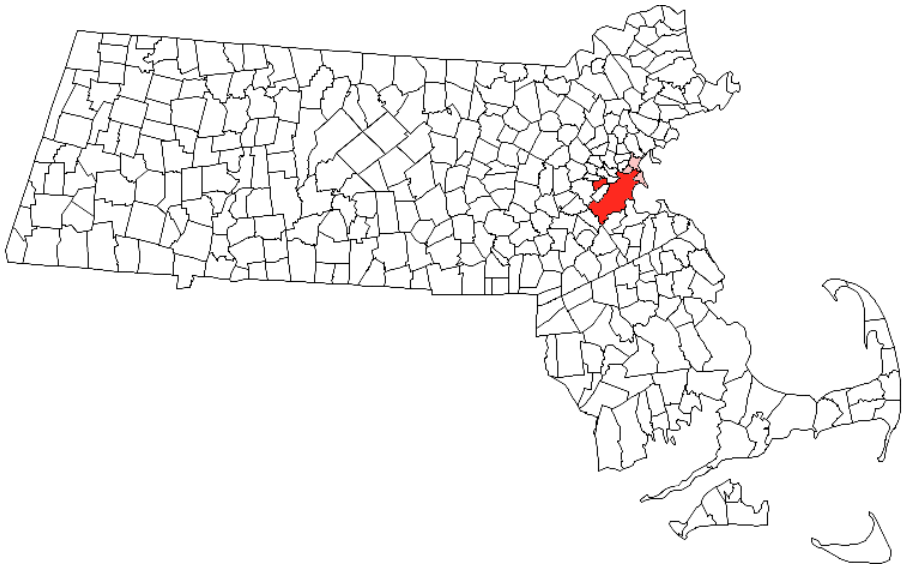
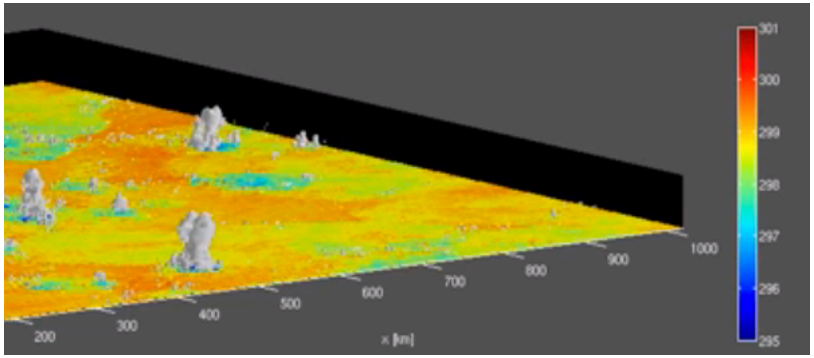
Linear Systems



Why model? To *understand*







http://www.youtube.com/watch?v=kLUzQjE8wU4
http://www.youtube.com/watch?v=chIzYtJjxhc